Measures for diffusion, ergodicity, & ageing

Ralf Metzler, U Potsdam & Wrocław U Science & Technology

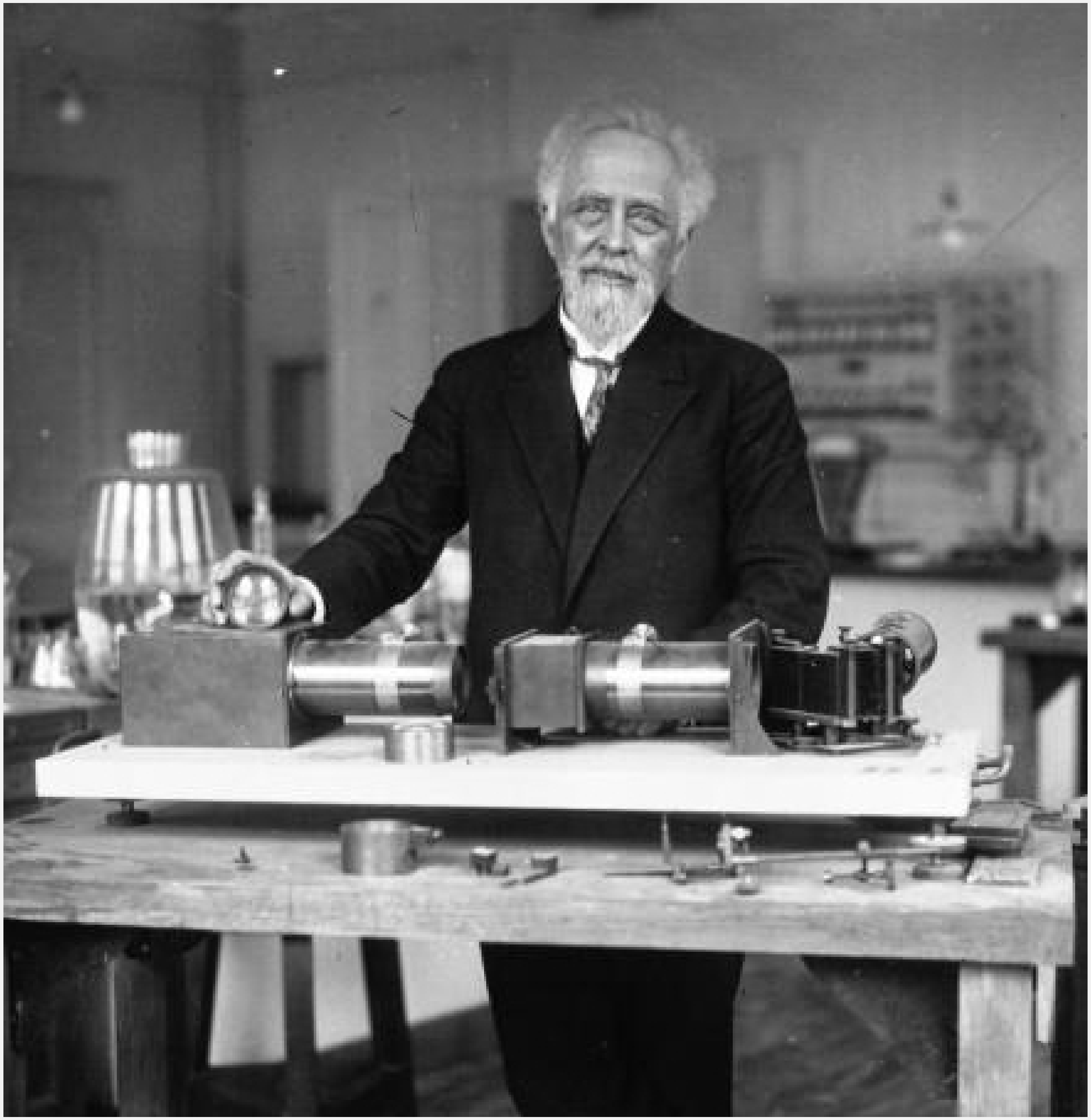
— Lanzhou, 4th August 2018 —

Microscopical observations

on the particles contained in the pollen of plants; and on the general existence of active molecules in organic and inorganic bodies

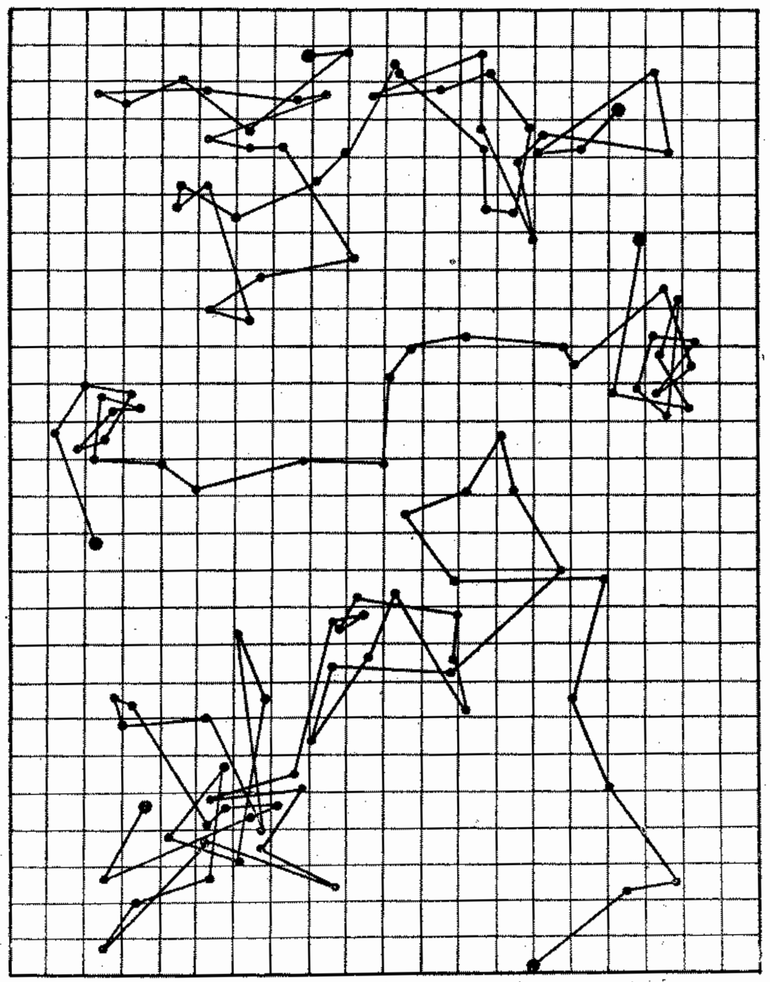
Rocks of all ages, including those in which organic remains have never been found, yielded the molecules in abundance. Their existence was ascertained in each of the constituent molecules of granite, a fragment of the Sphinx being one of the specimens examined.

R Brown, Edinburgh new Philosophical Journal 358-371 (1828)



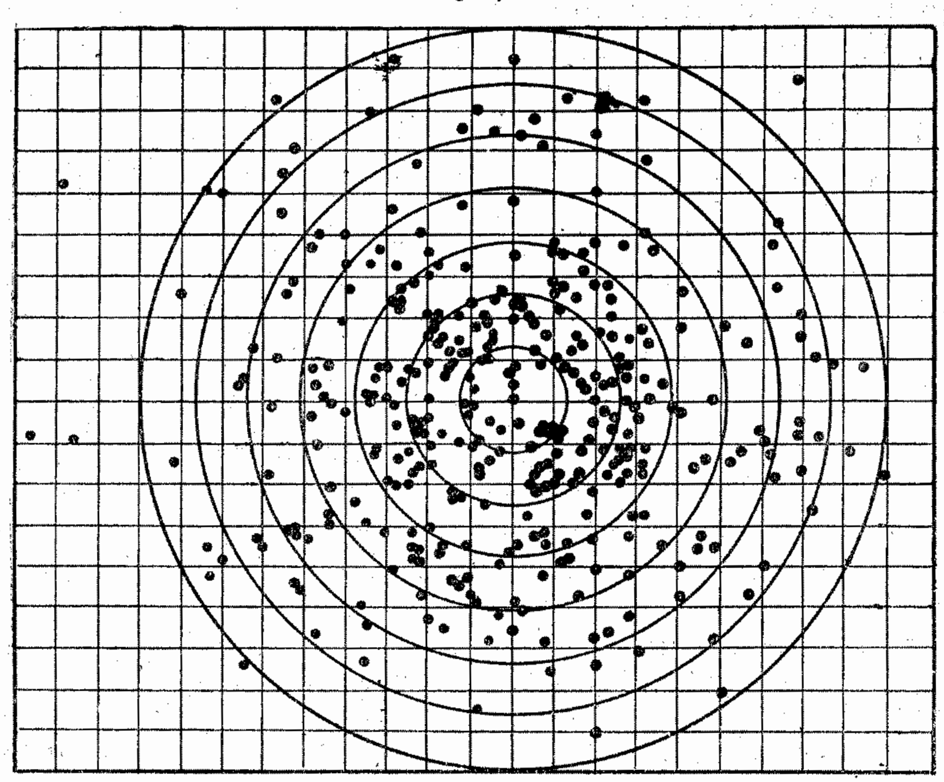
Brownian motion

Fig. 6.



Δt=30 sec

Fig. 7.

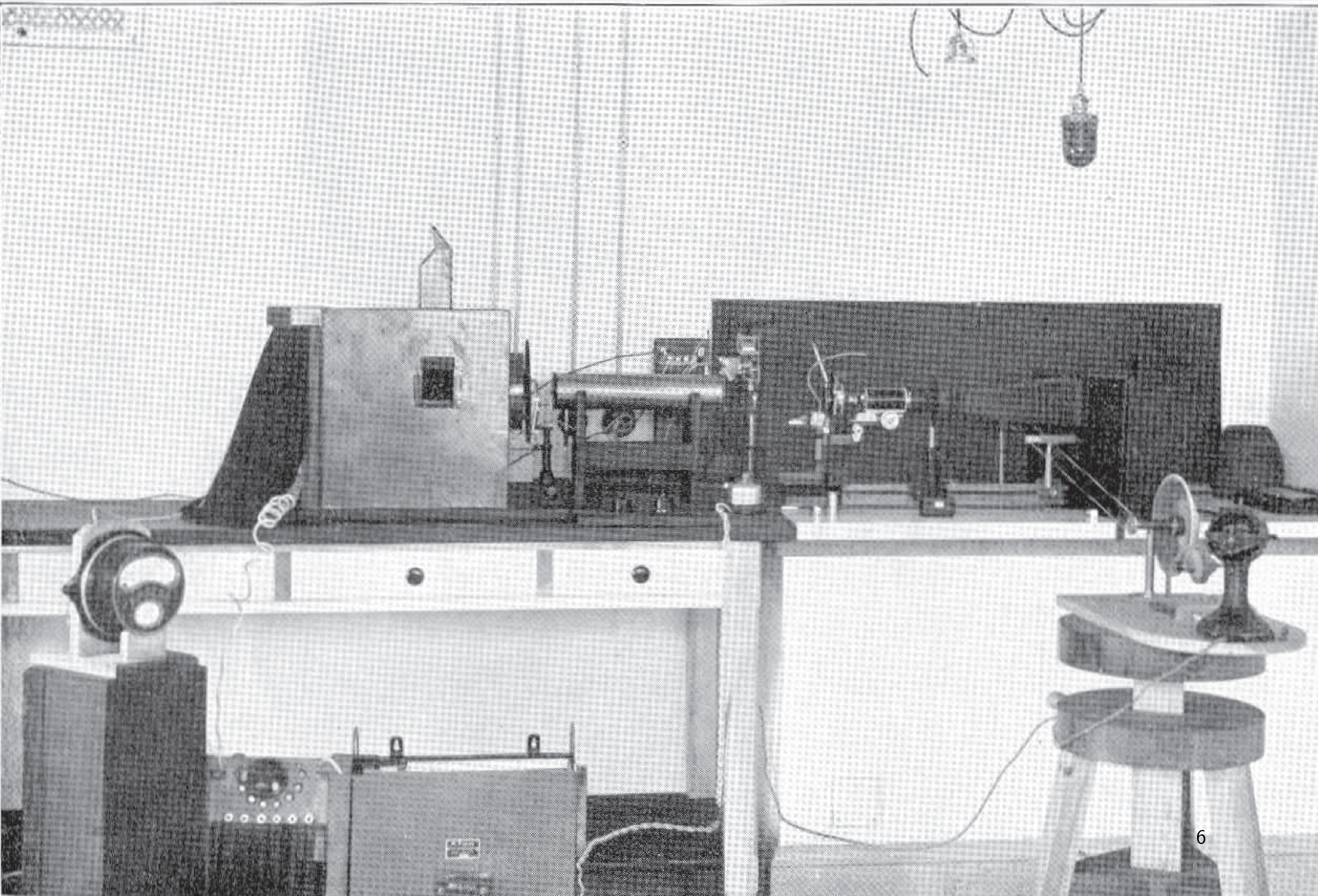


$$P(\mathbf{r}, \Delta t) = \frac{1}{(4\pi K \Delta t)^{d/2}} \exp\left(-\frac{r^2}{4K\Delta t}\right)$$

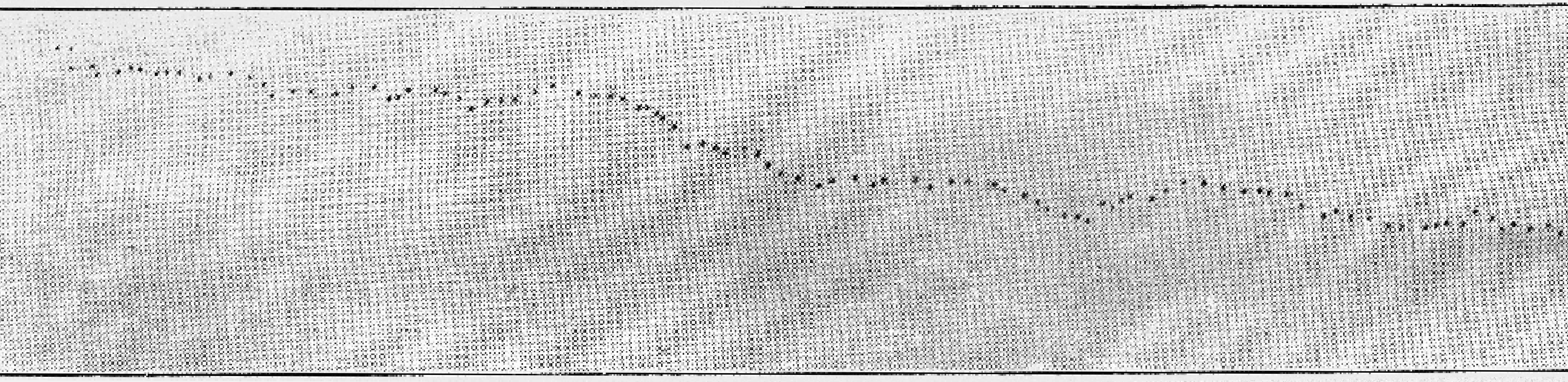
Einstein-Smoluchowski relation:

$$K = \frac{k_B T}{m\eta} = \frac{(R/N_A)T}{m\eta}$$

J Perrin, Comptes Rendus (Paris) 146 (1908) 967: $N_A = 70.5 \times 10^{22}$



Ivar Nordlund: 100+ years of SPT with time series analysis



Mercury droplet in aqueous solution

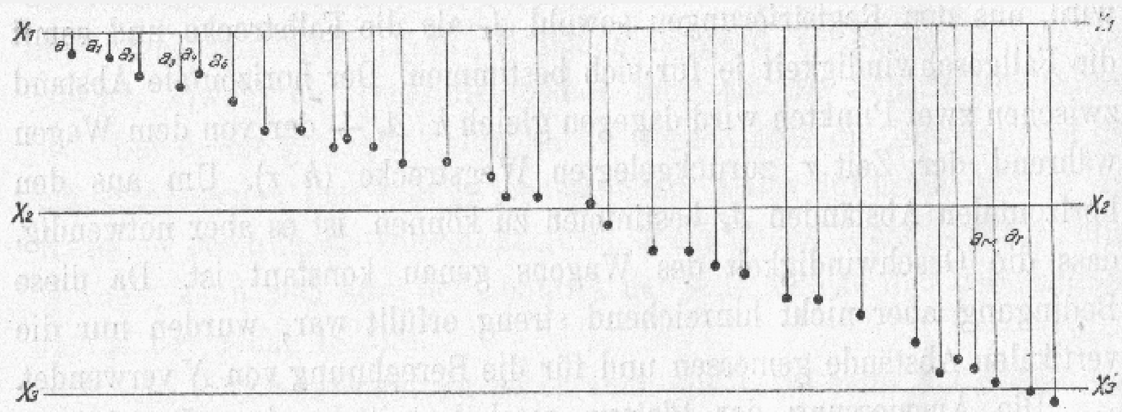


Fig. 5.

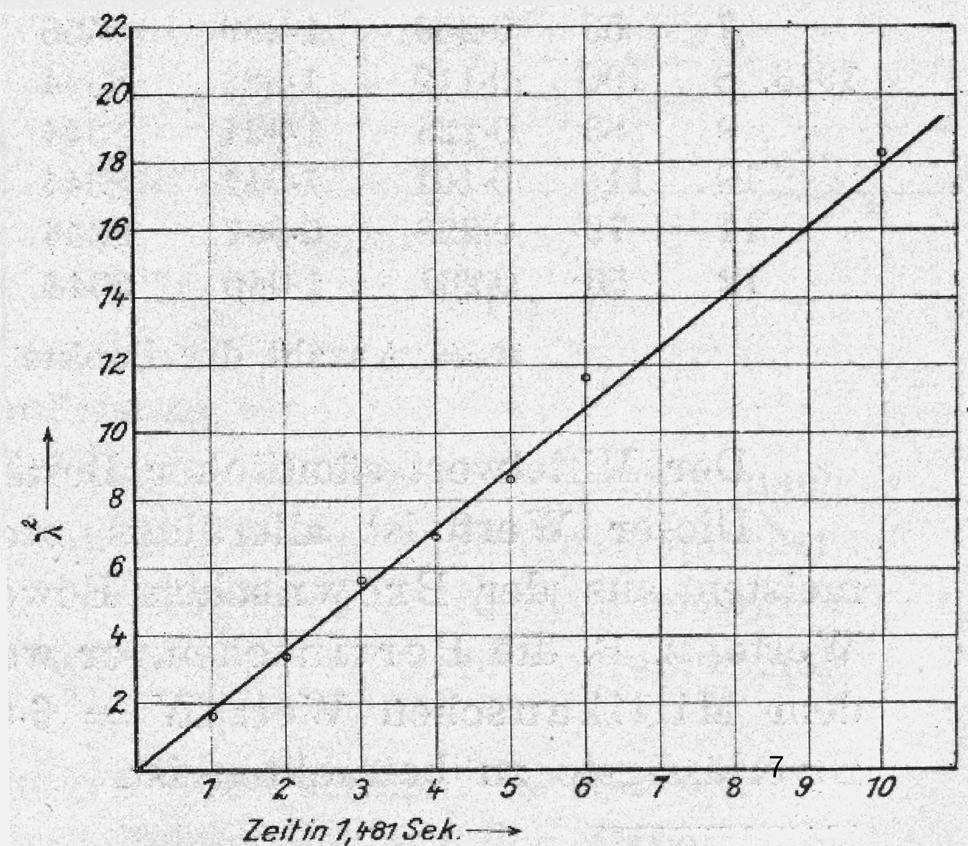


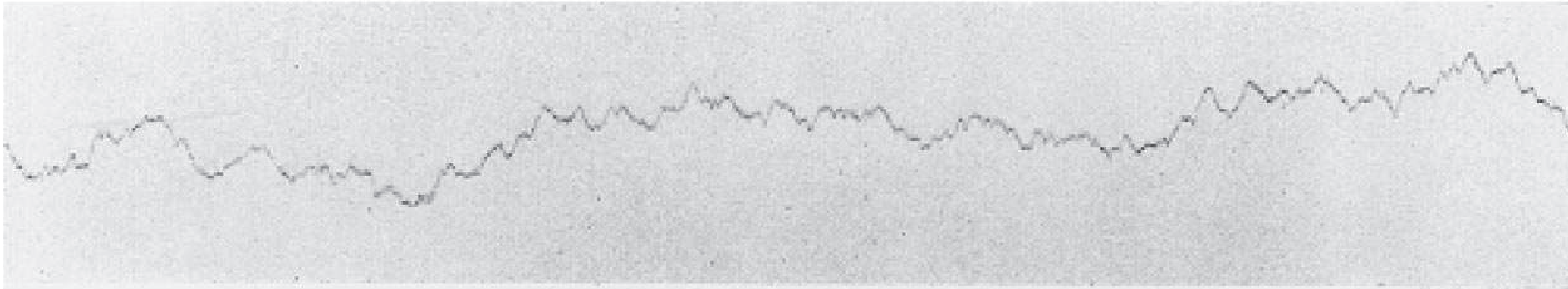
Fig. 11.

I Nordlund, Z Physik (1914): $N_A = 5.91 \times 10^{23}$

Eugen Kappler: ultimate diffusion measurements

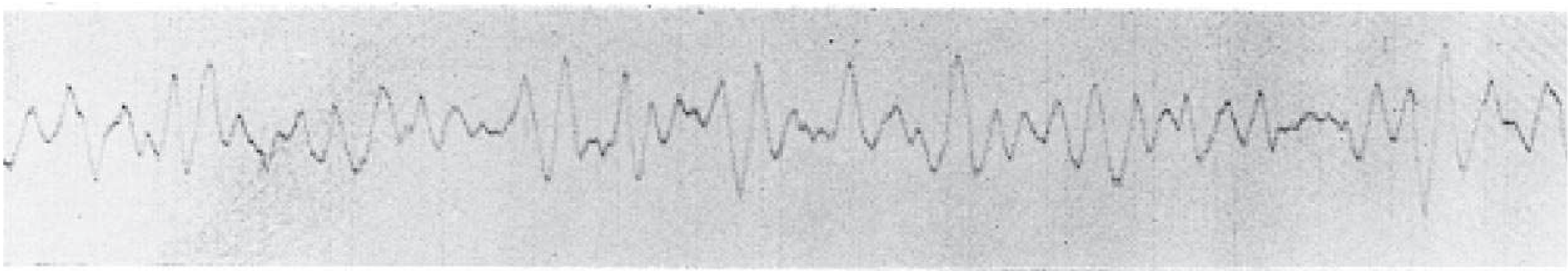
Obiturary by L Reimer, Physikalische Blätter Feb 1978 pp 86

Eugen Kappler: ultimate diffusion measurements



Registrieraufnahme der Brownschen Bewegung (natürliche Größe).
Direktionskraft 9,428 · 10^{-9} abs. Einh. Trägheitsmoment: 1 · 10^{-7} abs. Einh. Abstand Spiegel–Kamera: 72,1 cm.
Zeitmarke: 30 sec $dx = 1$ mm. a) Atmosphärendruck. Temperatur 13° C

Fig. 5a

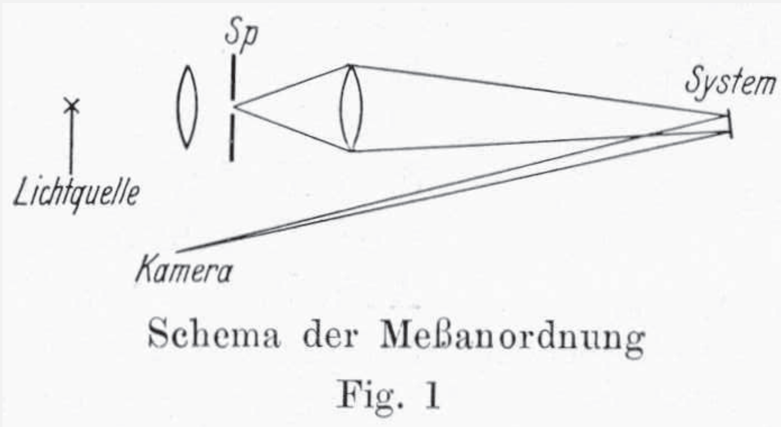


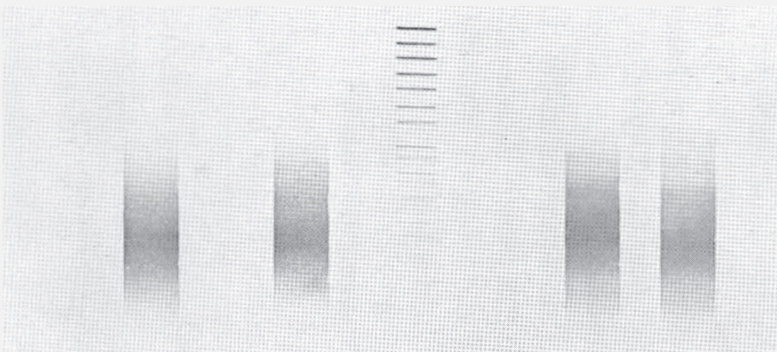
Registrieraufnahme der Brownschen Bewegung (natürliche Größe).
Direktionskraft 9,428 · 10^{-9} abs. Einh. Trägheitsmoment 1 · 10^{-7} abs. Einh. Abstand Spiegel–Kamera: 72,1 cm.
Zeitmarke: 30 sec $dx = 1$ mm. b) 1 · 10^{-3} mm Hg. Temperatur 13° C

Fig. 5b

E Kappler, Ann d Physik (1931): $N_A = 60.59 \times 10^{22} \pm 1\%$

Brownian motion & Kappler's diffusion measurements

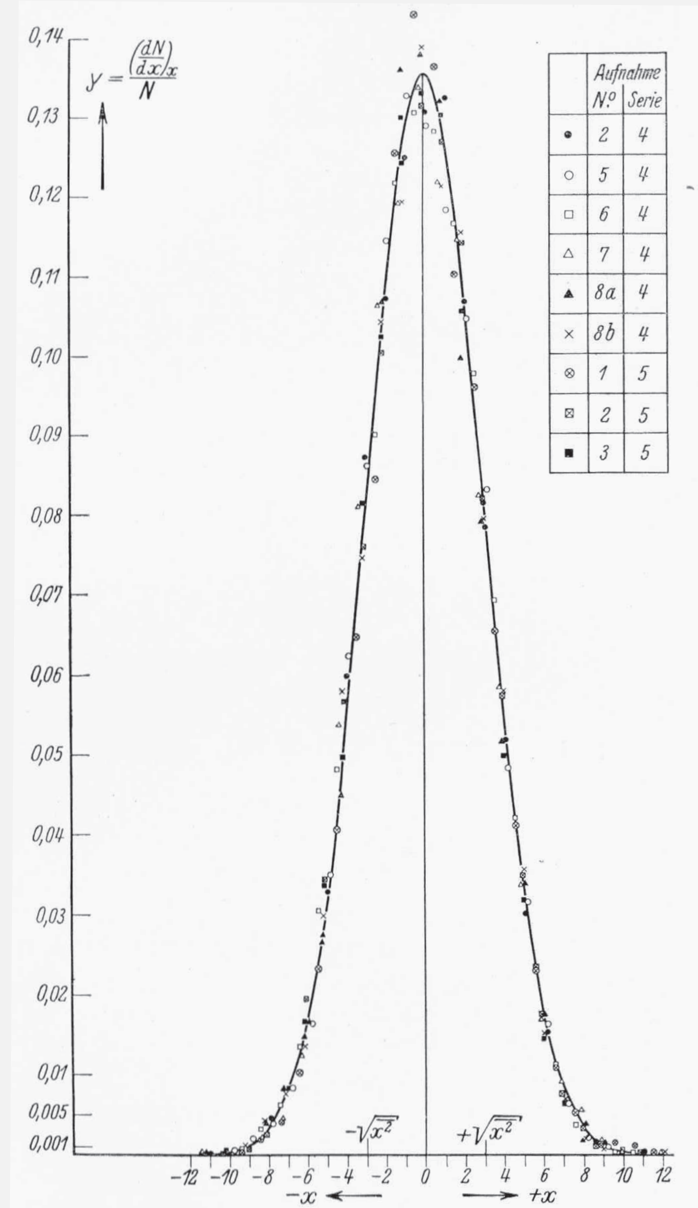




Distribution @ fixed photographic plate

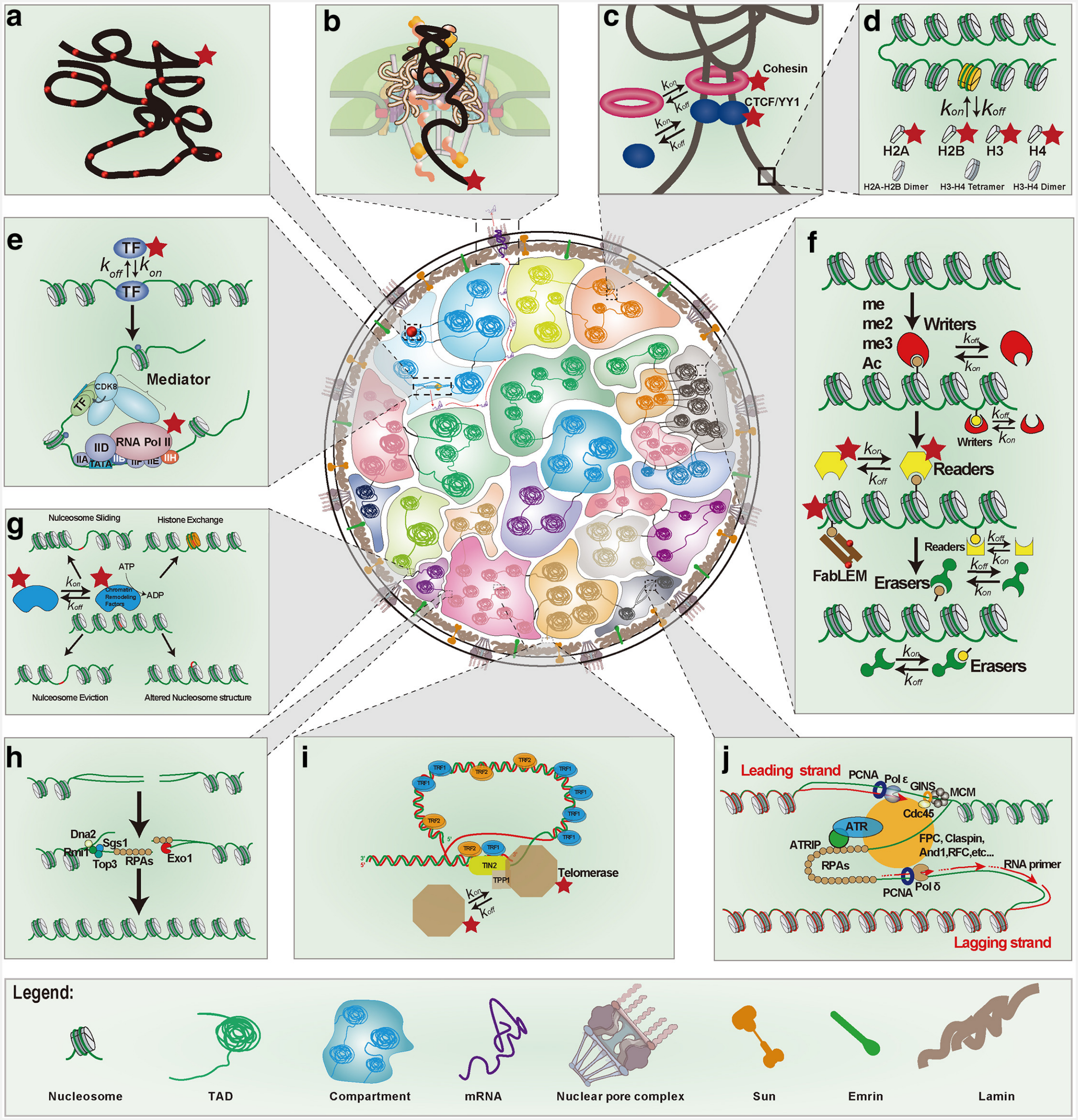
$$\langle \mathbf{r}^2(t)\rangle = 2dKt$$

$$P(\mathbf{r},t) = (4\pi Kt)^{-d/2}\exp(-\mathbf{r}^2/[4Kt])$$

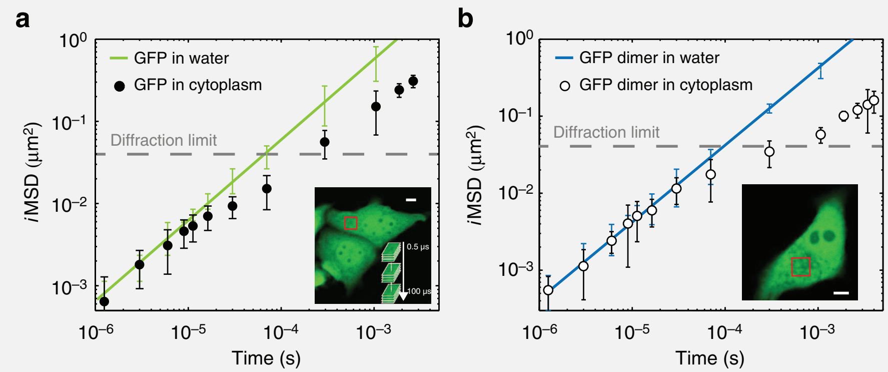


E Kappler, Ann d Physik (1931): $N_A = 60.59 \times 10^{22} \pm 1\%$

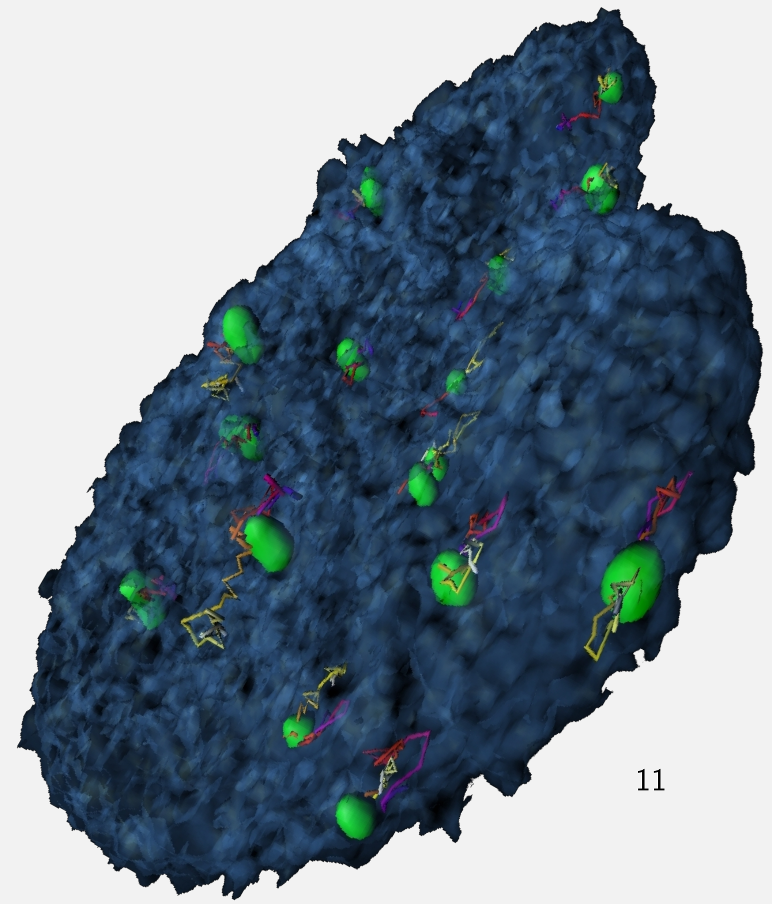
Single molecule imaging, state-of-the-art



S Shao, B Xue & Y Sun, Biophys J (2018)

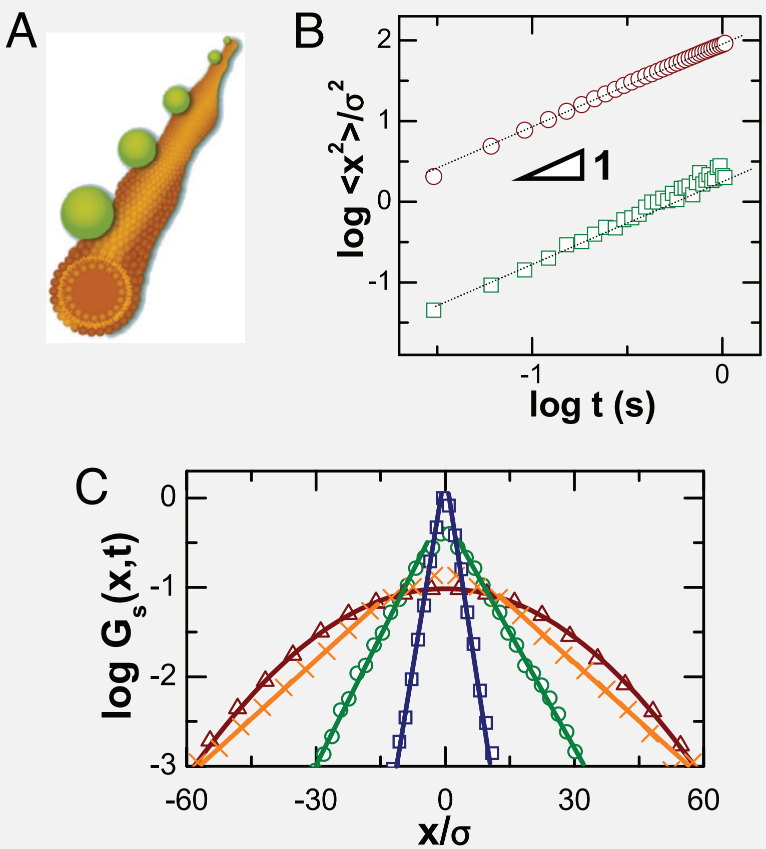


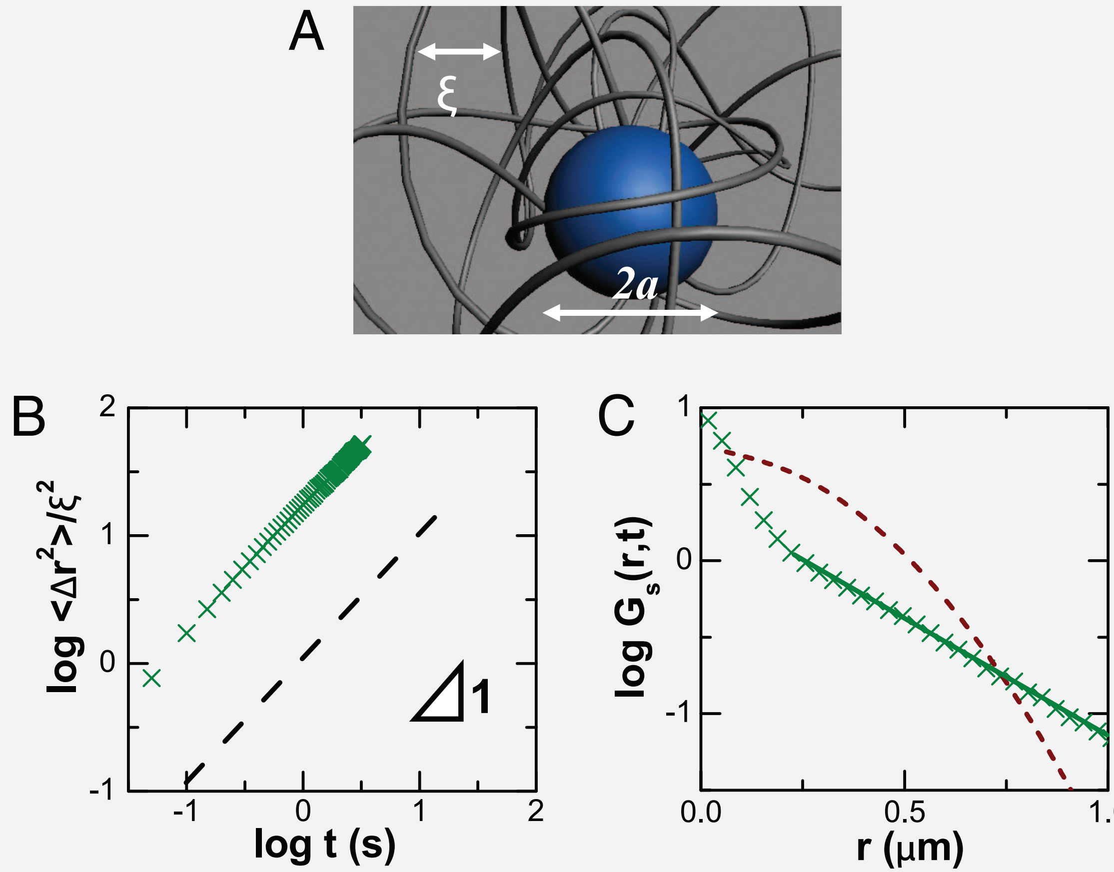
C di Renzo et al, Nat Comm (2014)



Courtesy Yuval Garini

When Brownian diffusion is not Gaussian



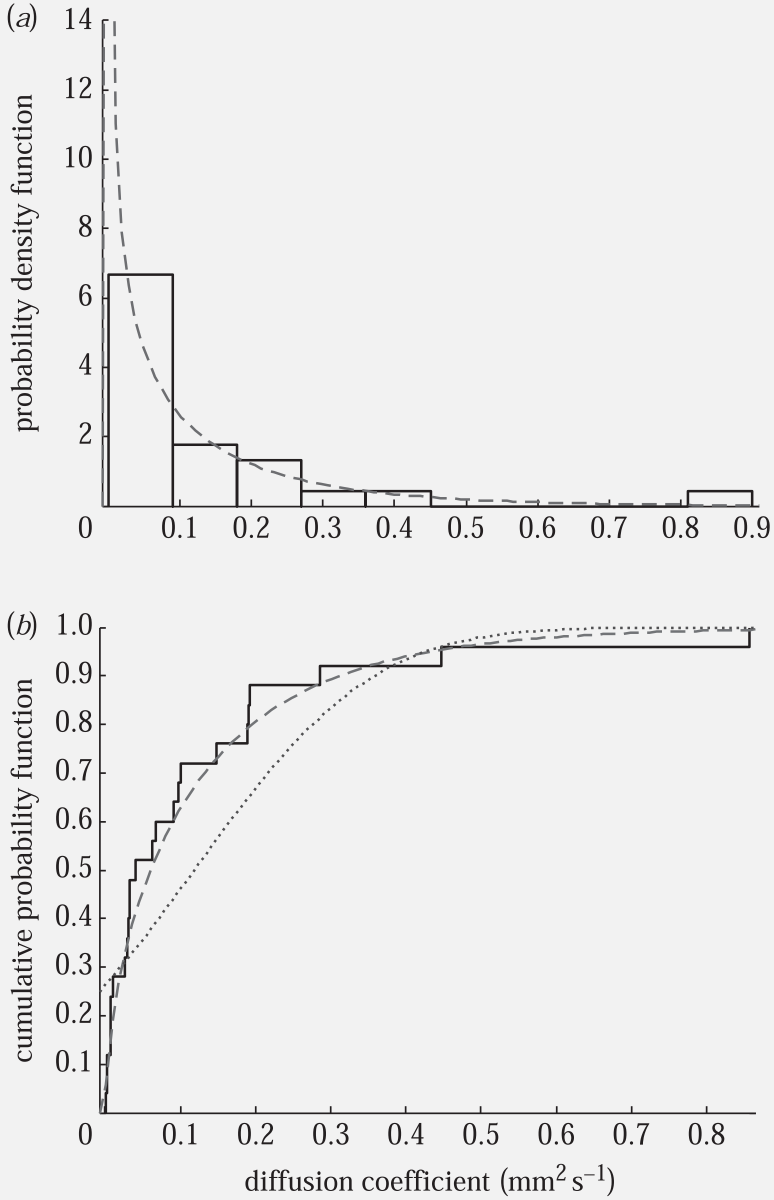


Colloidal beads diffusing on nanotubes

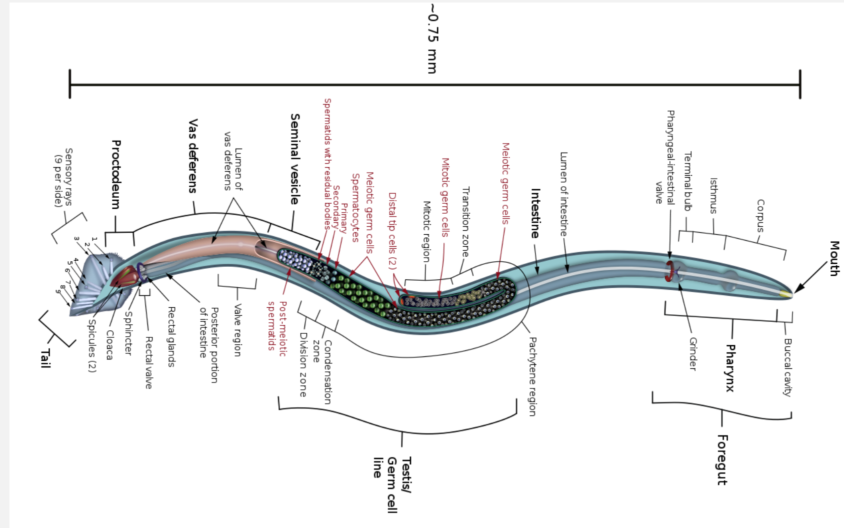
Nanospheres diffusing in entangled actin

Wang et al, PNAS (2009); Nature Mat (2012)

Heterogeneous diffusion in population of nematodes



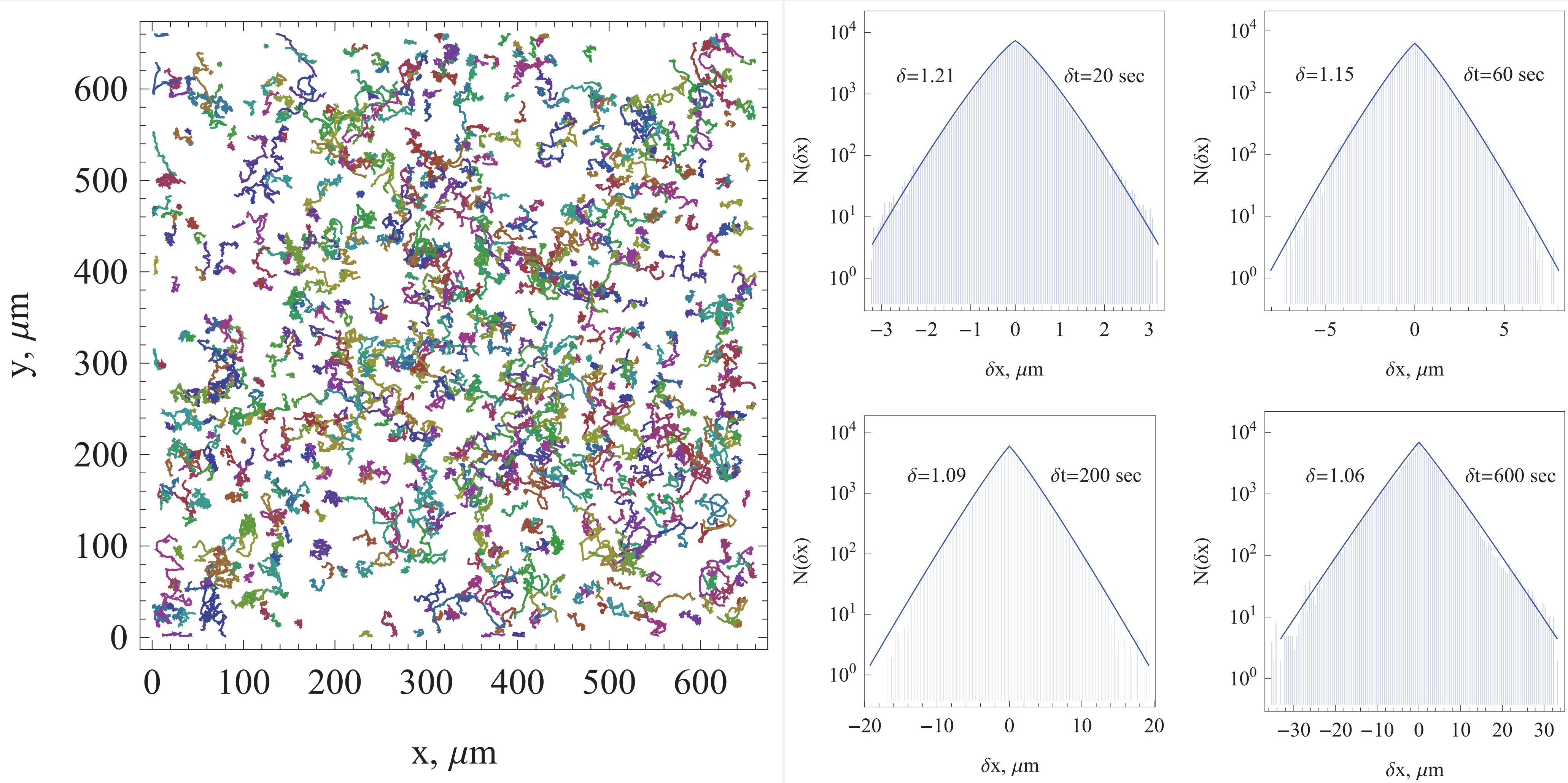
S Hapca, JW Crawford & IM Young, Roy Soc Interface (2009)



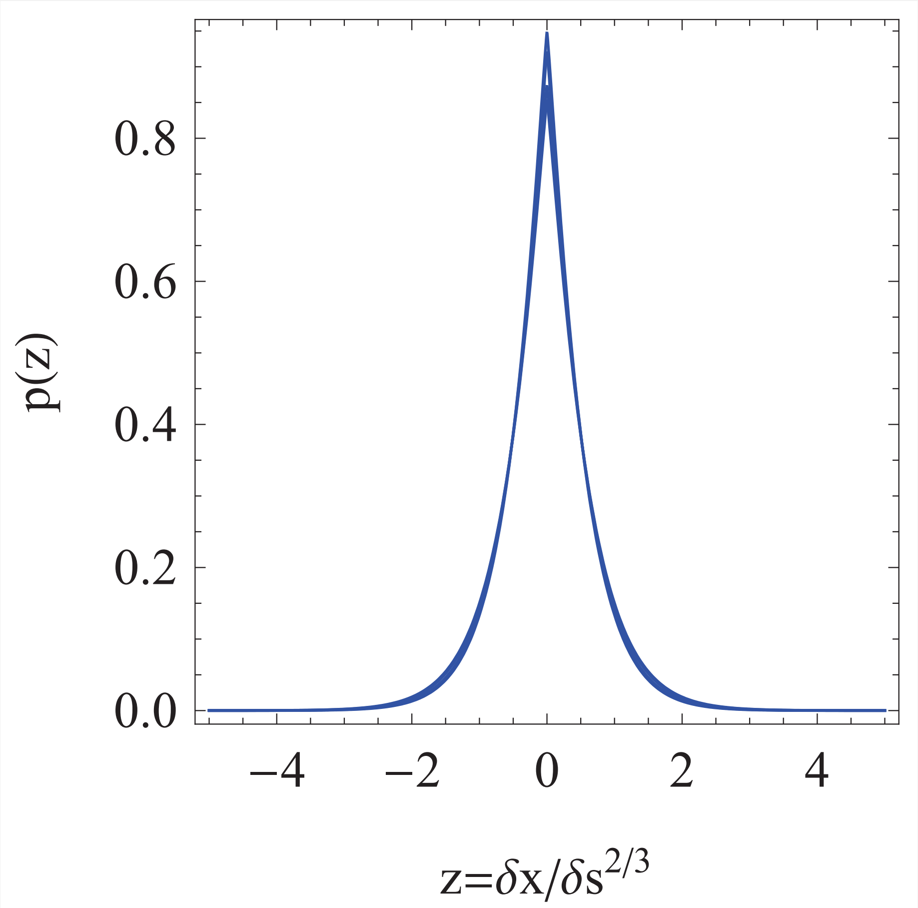
Male C. elegans nematode

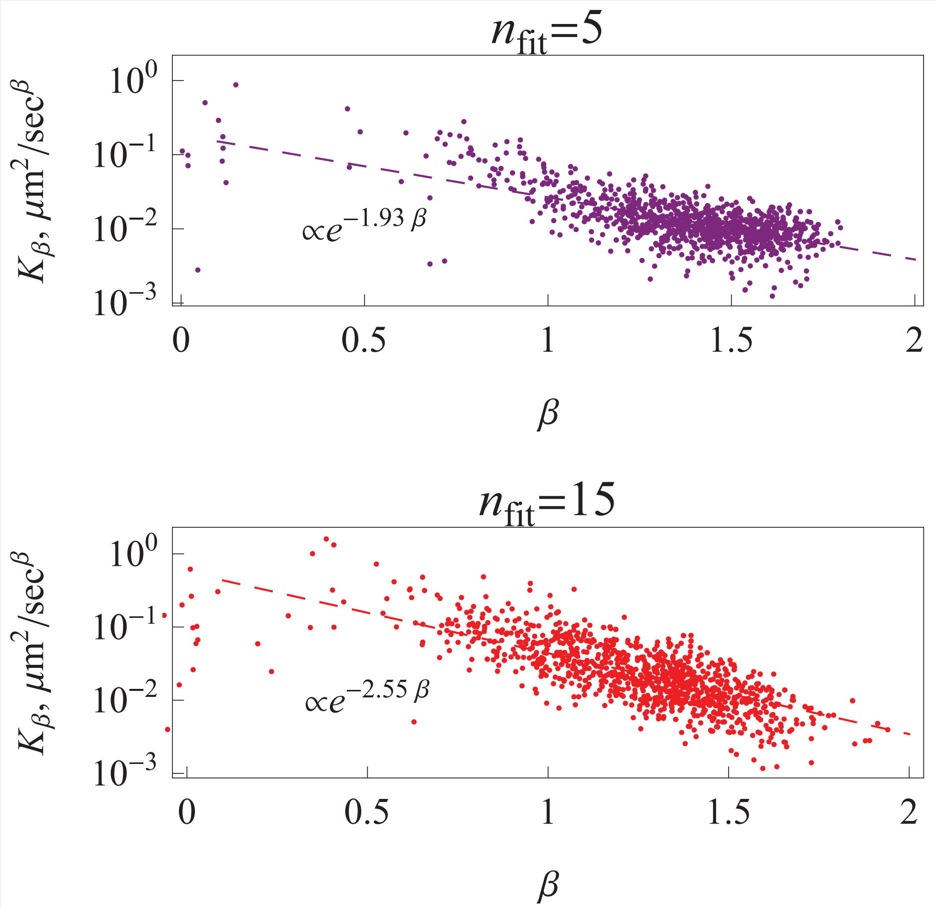
Soybean cyst nematode & egg

Non-Gaussian diffusion of Dictyostelium cells



Non-Gaussian diffusion of Dictyostelium cells





Similarity function

$$K_\beta \simeq \exp(-c_1\beta + c_2)$$

AG Cherstvy, O Nagel, C Beta & RM (2018)

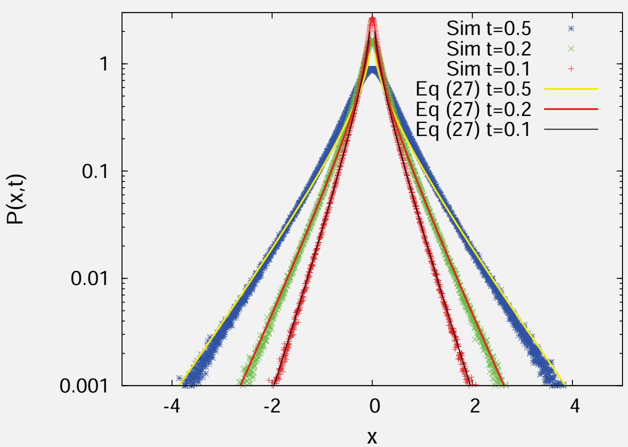
Fickian, non-Gaussian diffusion with diffusing diffusivity

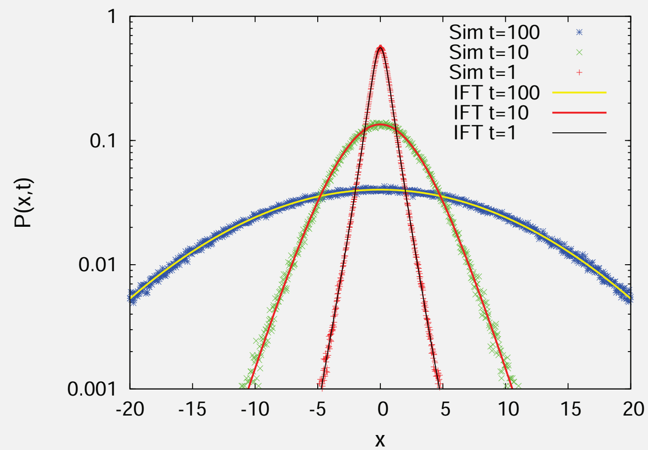
B Wang, J Kuo, SC Bae & S Granick, Nat Mat (2012): $\langle x^2(t)\rangle = 2K_1 t$, yet $P(x,t)$ non-Gaussian. Superstatistical approach $P(x,t) = \int_0^\infty G(x,t)p(D)dD$

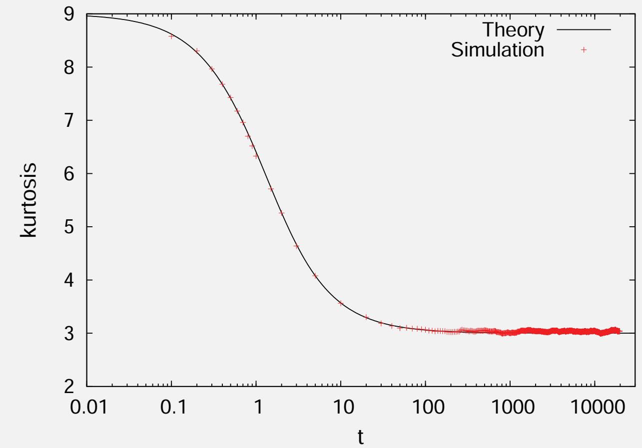
MV Chubinsky & G Slater, PRL (2014); R Jain & KL Sebastian, JPC B (2016): diffusing diffusivity

Our minimal model for diffusing diffusivity:

$$\dot{x}(t) = \sqrt{2D(t)}\xi(t)$$
$$D(t) = y^2(t)$$
$$\dot{y}(t) = -\tau^{-1}y + \sigma\eta(t)$$

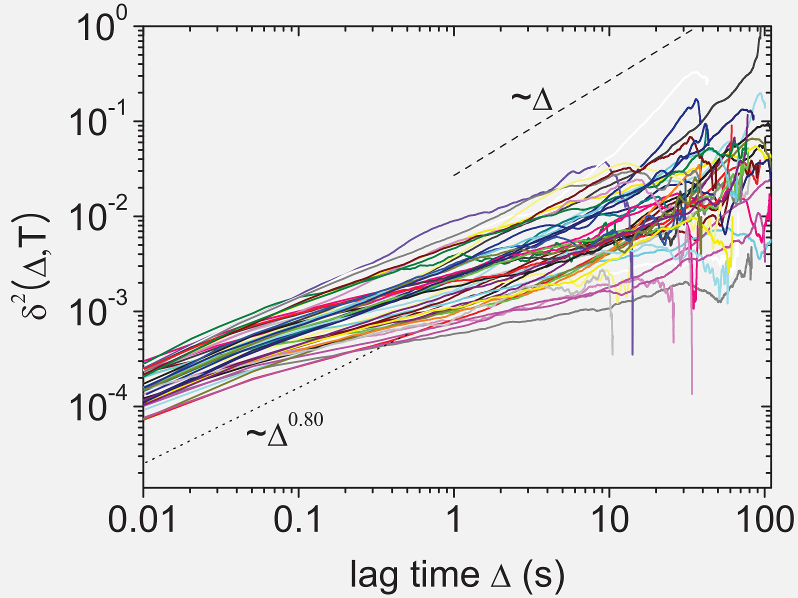


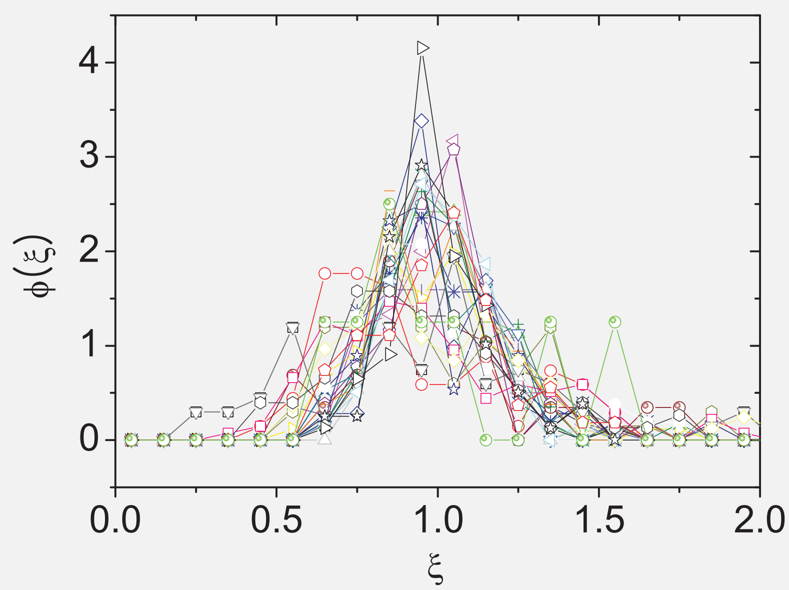




AV Chechkin, F Seno, RM & IM Sokolov, PRX (2017); generalised $\gamma(D)$: V Sposini, AV Chechkin, G Pagnini, F Seno & RM, NJP (2018)

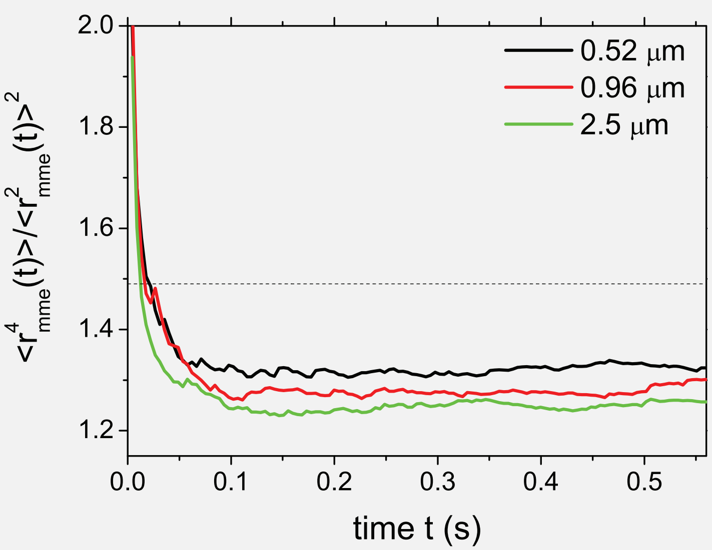
Passive motion of submicron tracers in cells is viscoelastic

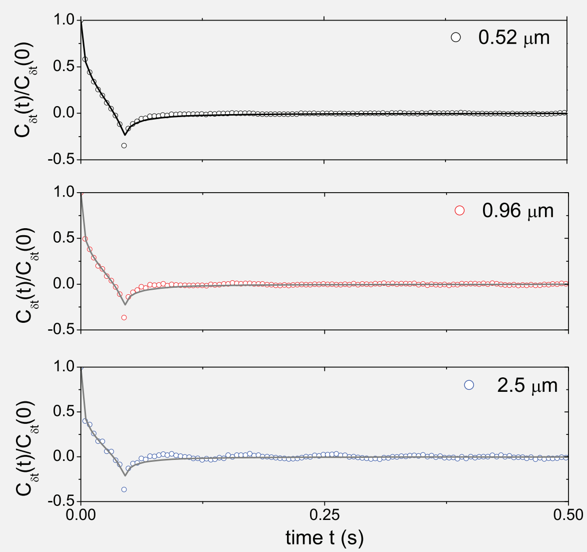


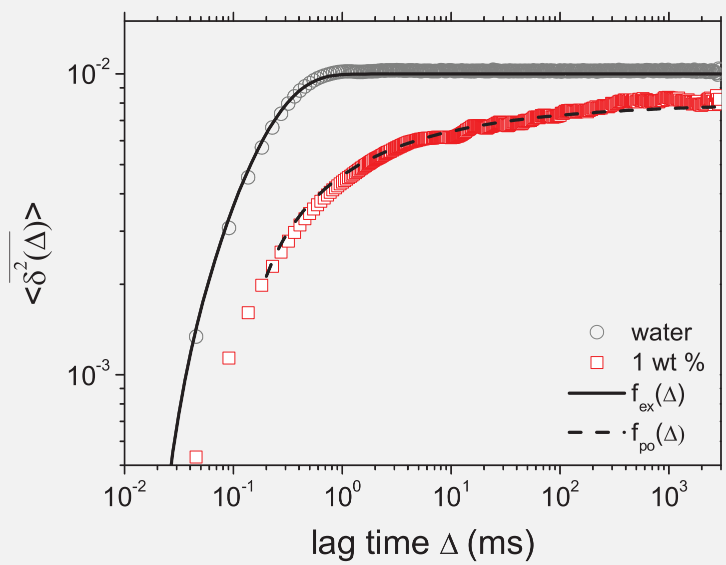


Lipid granules in living yeast cells ←

Tracer beads in wormlike micellar solution ↓

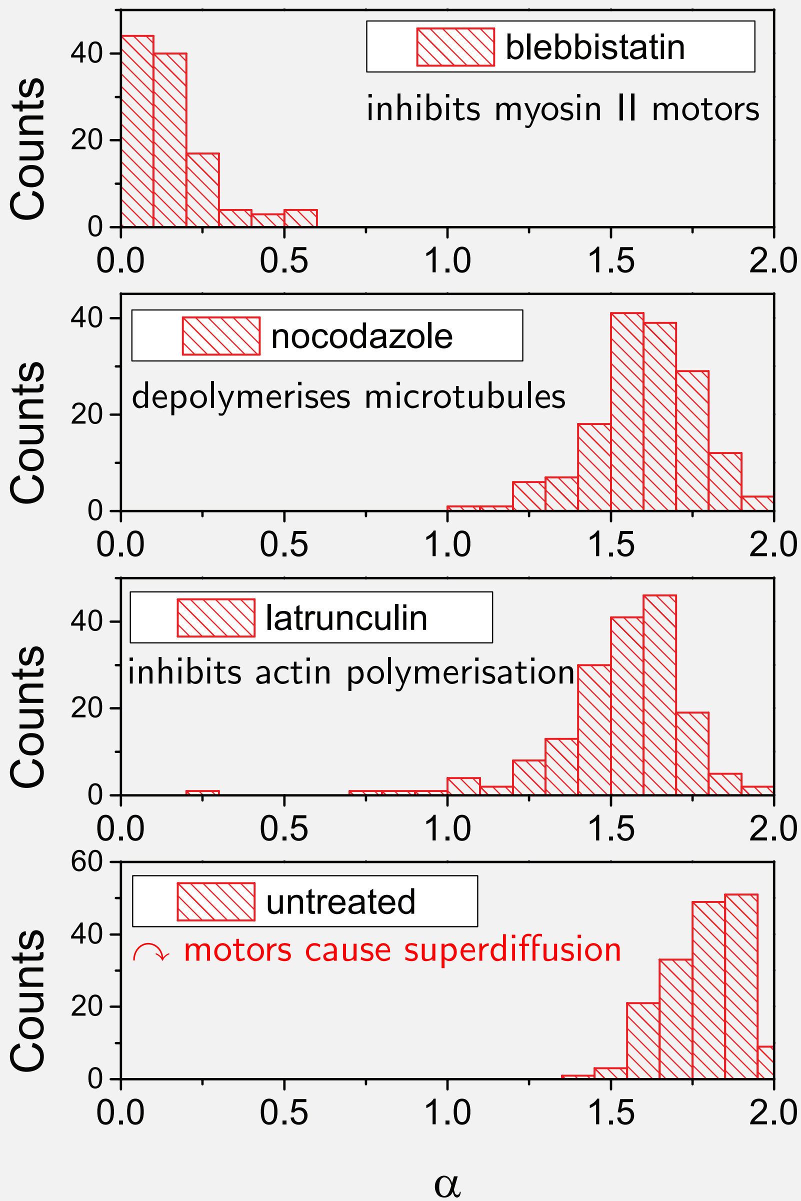


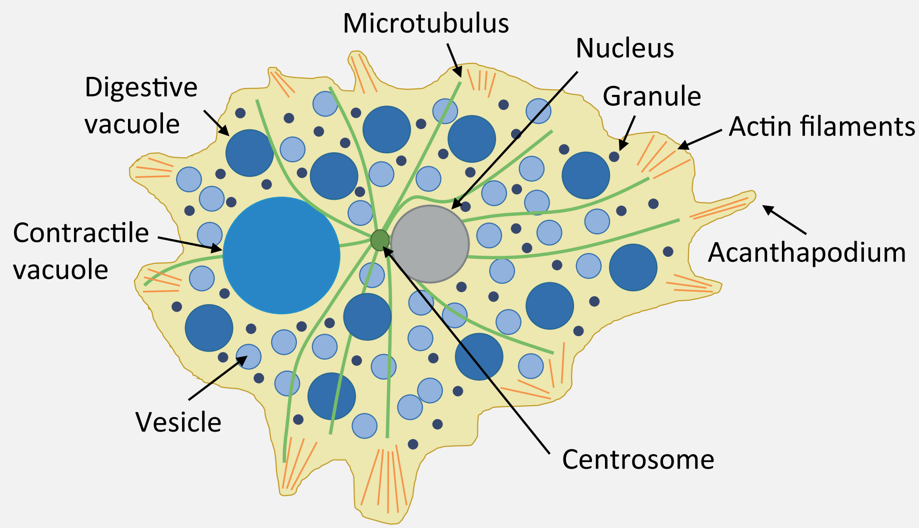


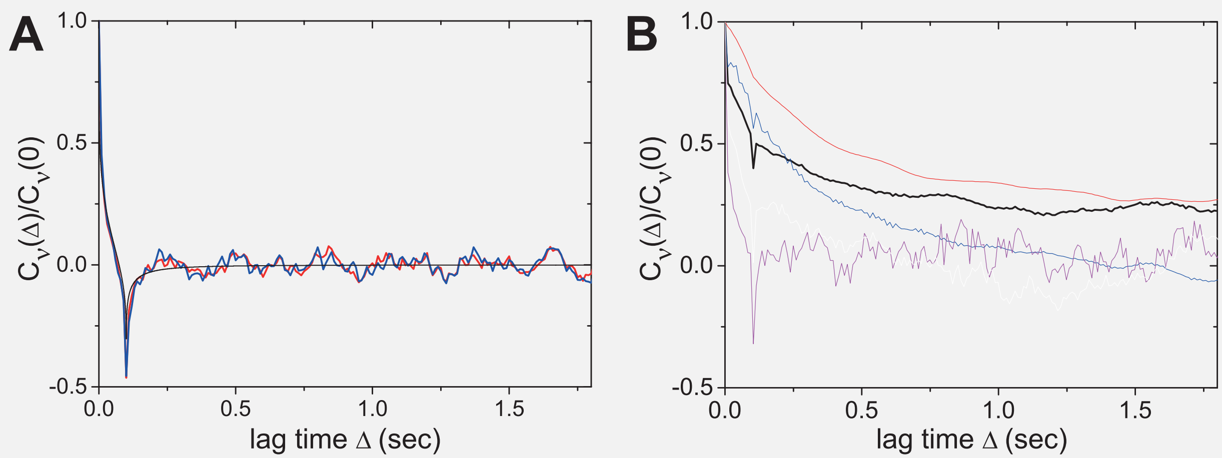


JH Jeon, . . . L Oddershede & RM, PRL (2011); JH Jeon, N Leijnse, L Oddershede & RM, NJP (2013)

Superdiffusion in supercrowded Acanthamoeba castellani



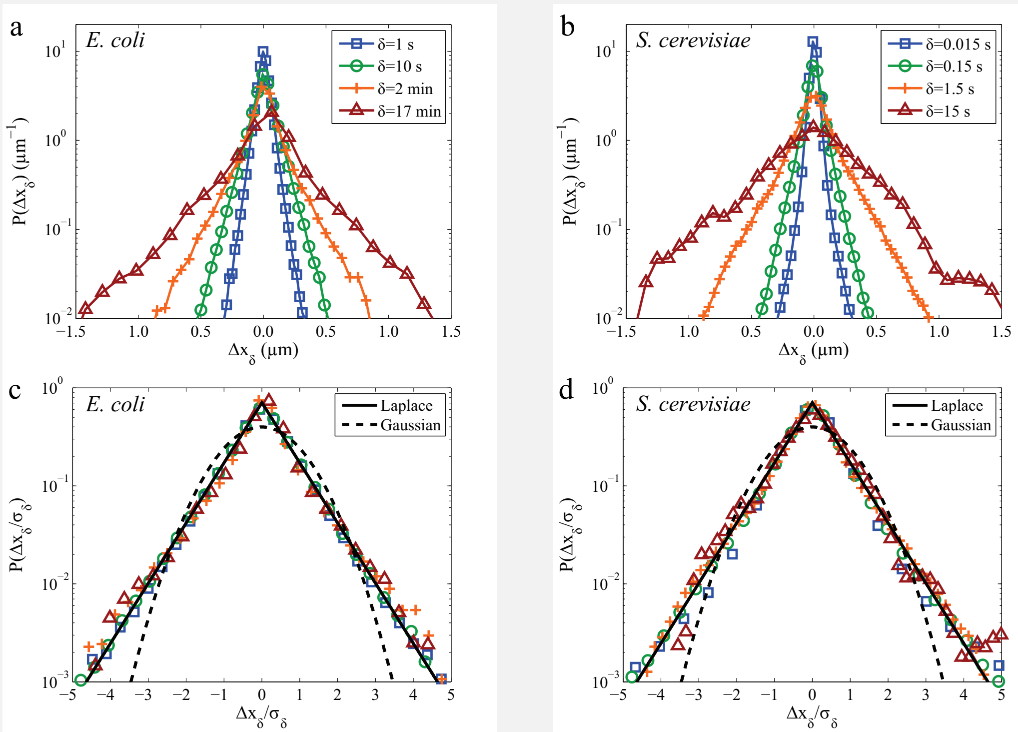


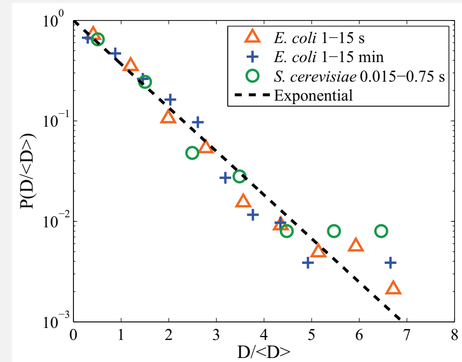


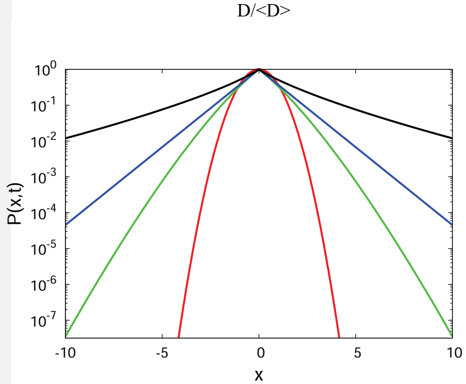
JF Reverey, J-H Jeon, H Bao, M Leippe, RM & C Selhuber-Unkel, Sci Rep (2015)

Non-Gaussian diffusion in viscoelastic systems

So far consensus: submicron tracer motion in cytoplasm is FBM-like, i.e., Gaussian
RNA-protein particles in E.coli & S.cerevisiae perform exponential anomalous diffusion:



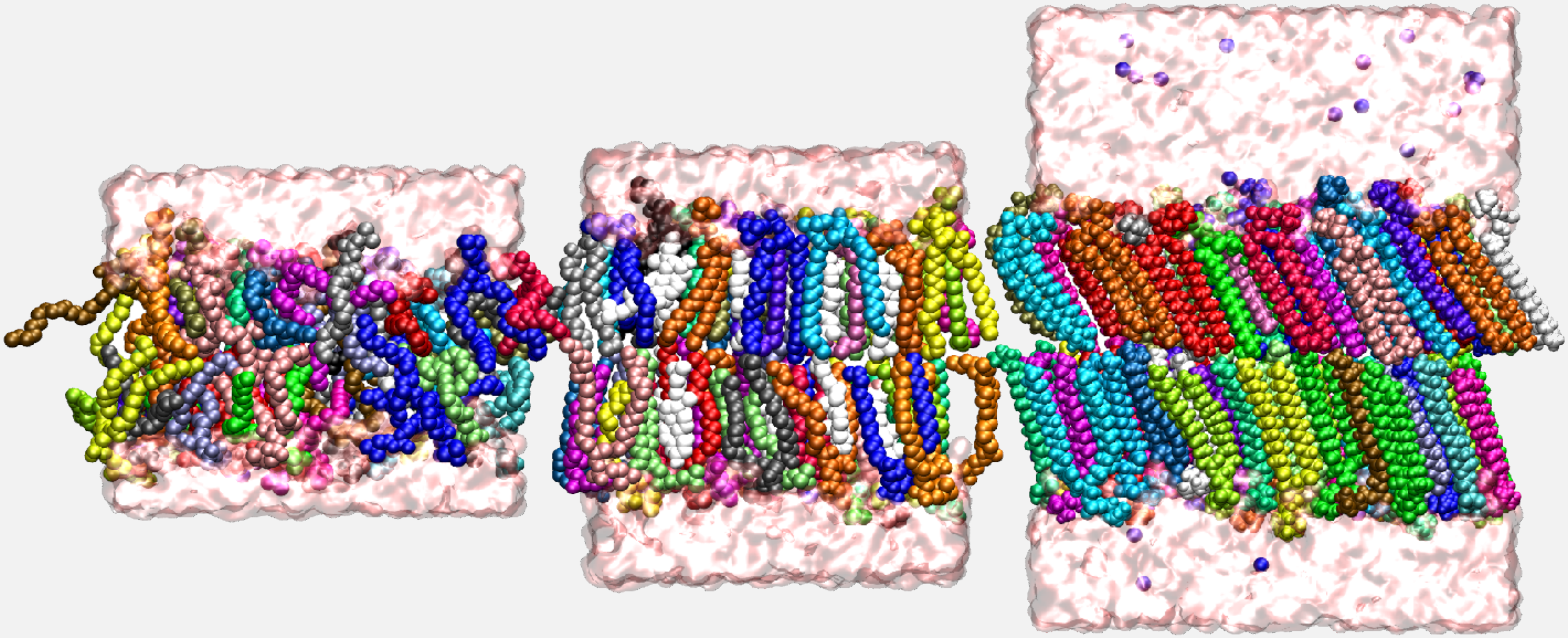




Modelling based on grey GLE: J Ślęzak, RM & M Magdziarz, NJP (2018)

TJ Lampo, S Stylianidou, MP Backlund, PA Wiggins & AJ Spakowitz, BPJ (2017); N&V: RM, BPJ (2017)

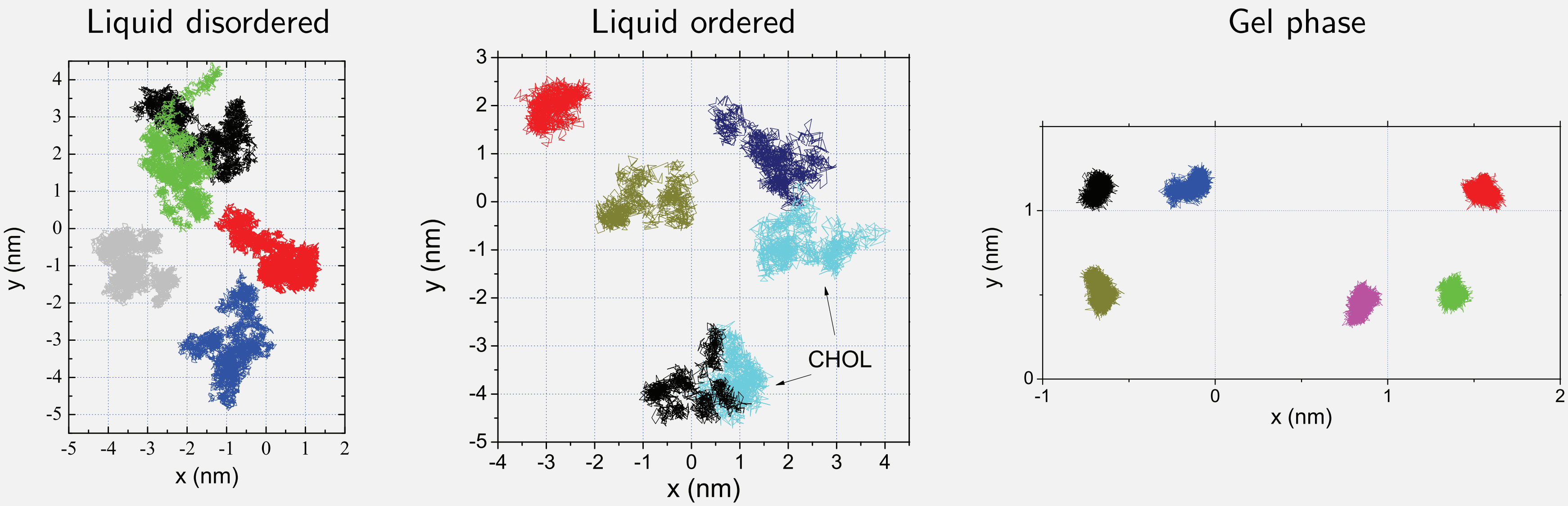
Single lipid motion in bilayer membrane MD simulations



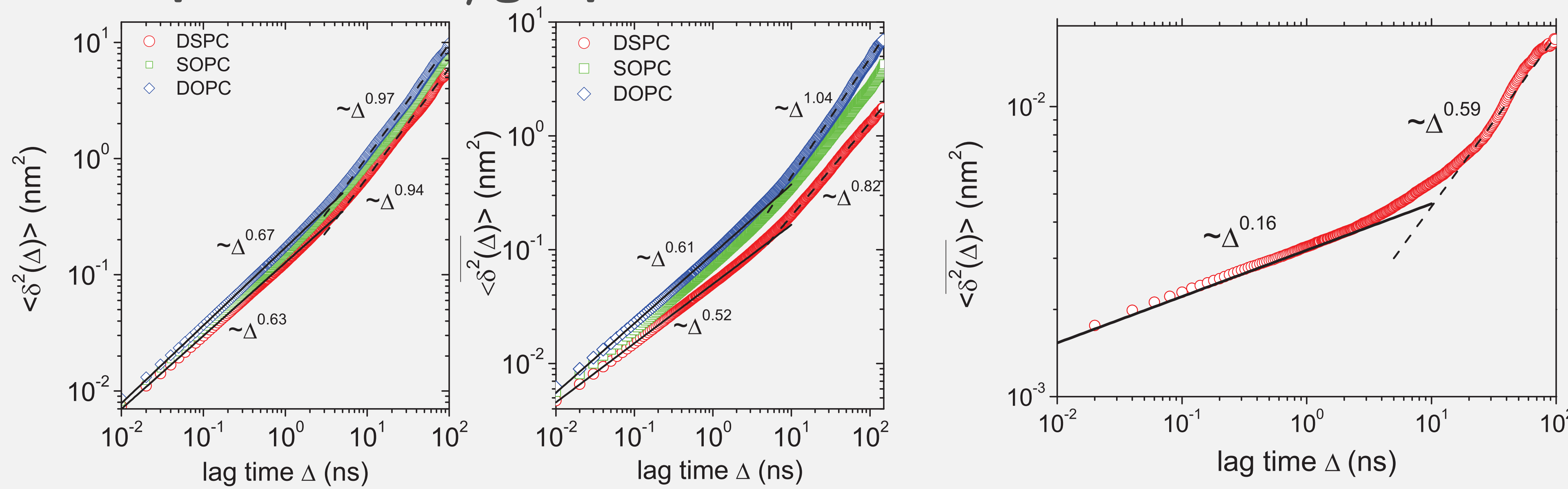
Liquid disordered   Liquid ordered   Gel phase

J-H Jeon, H Martinez-Seara Monne, M Javanainen & RM, PRL (2012)

Sample trajectories for the lipid & cholesterol motion



Liquid ordered/gel phases: extended anomalous diffusion

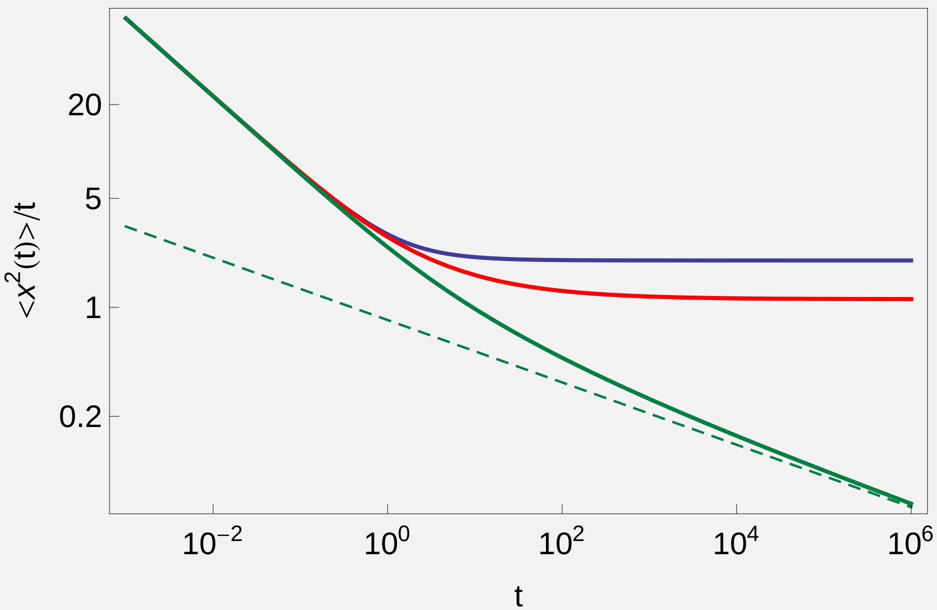


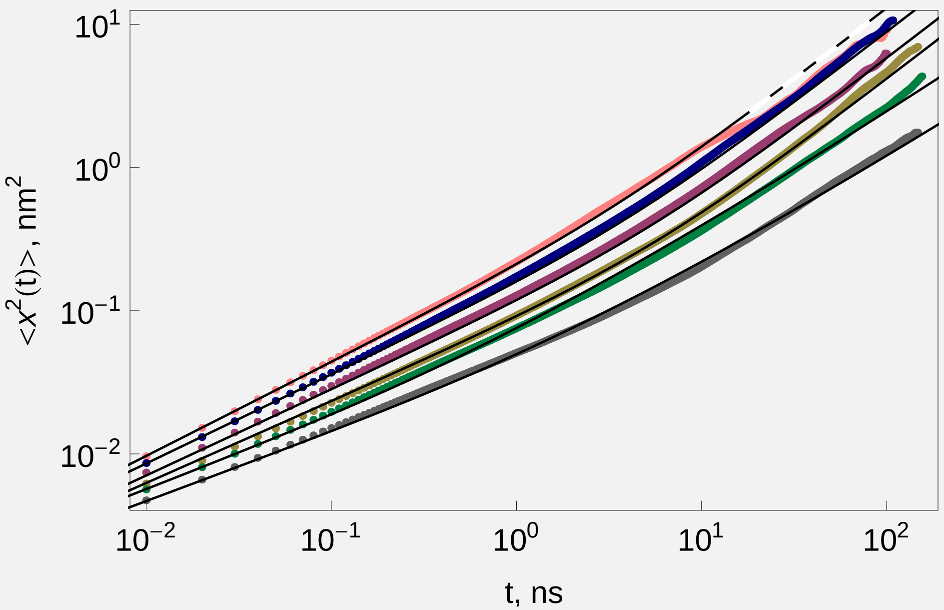
J-H Jeon, H Martinez-Seara Monne, M Javanainen & RM, PRL (2012)

Tempered FBM & FLE motion: sub- to normal diffusion

Consider tempered fGn:

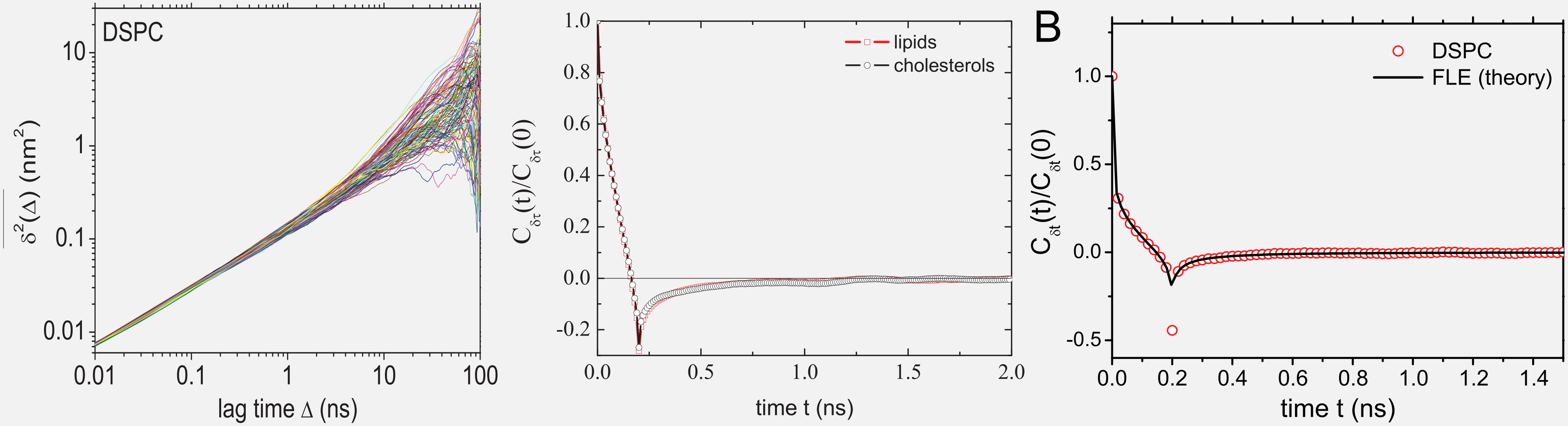
$$\langle \xi(t)\xi(t+\tau)\rangle = \begin{cases} \dfrac{C}{\Gamma(2H-1)}\tau^{2H-2}e^{-\tau/\tau_\star} \\ \dfrac{C}{\Gamma(2H-1)}\tau^{2H-2}\left(1+\dfrac{\tau}{\tau_\star}\right)^{-\mu} \end{cases}$$



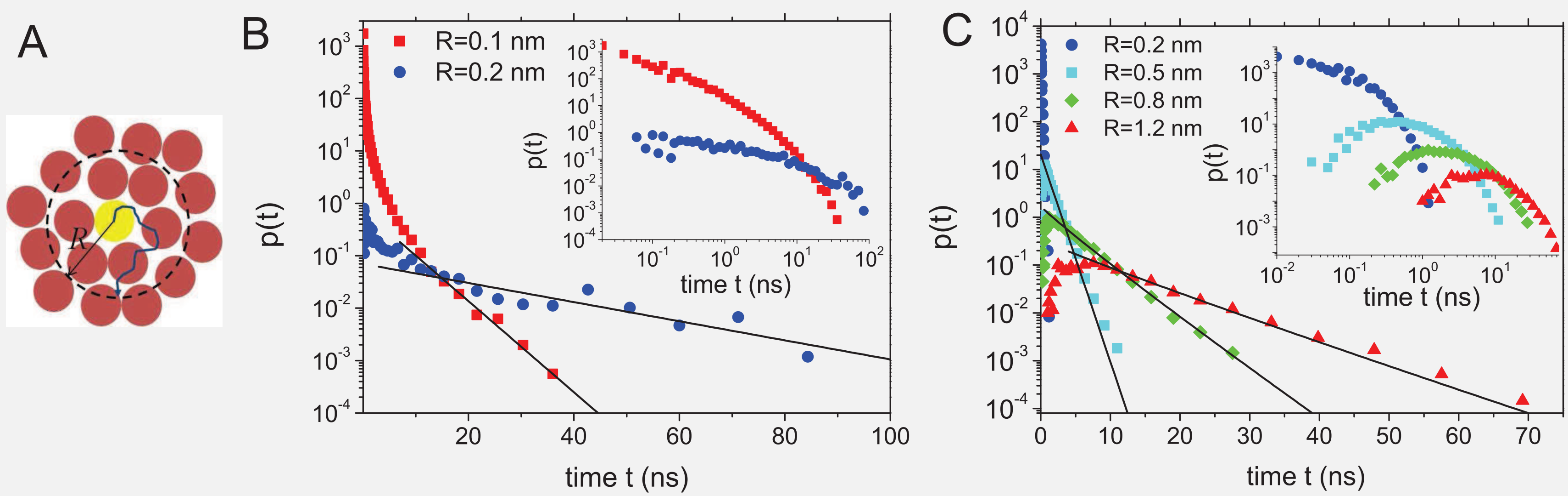


D Molina-Garcia, T Sandev, G Pagnini, AV Chechkin & RM (2018)

Reproducible TA MSD & antipersistent correlations

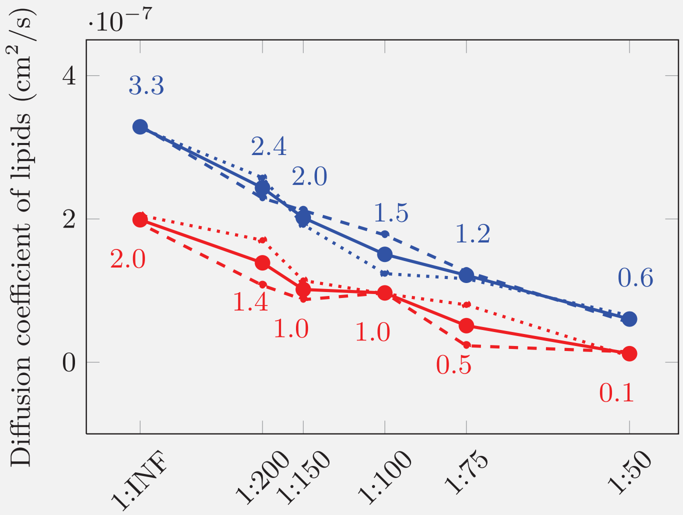


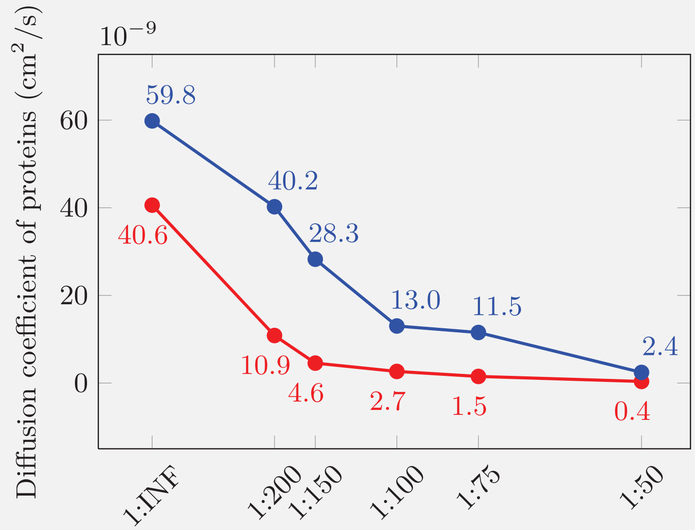
Rattling dynamics: exptl first passage PDF ↷ FLE motion



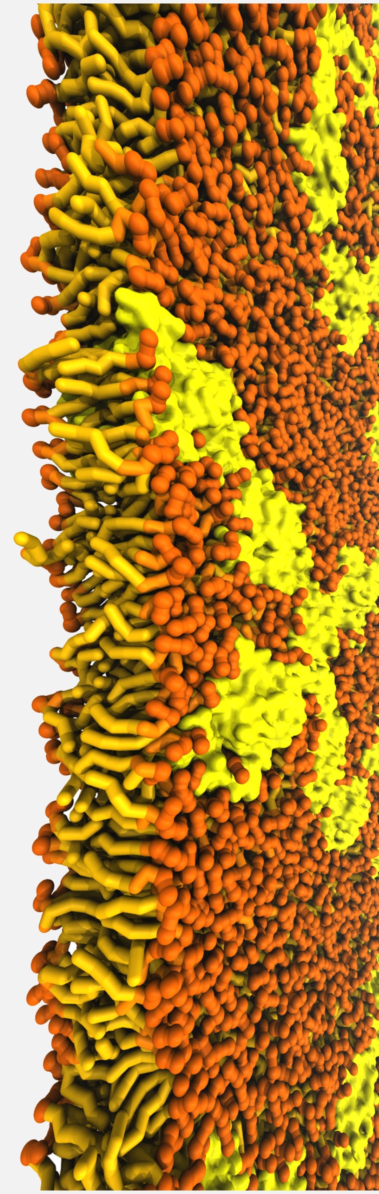
D Molina-Garcia, T Sandev, G Pagnini, AV Chechkin & RM (2018)

Protein crowded membranes reduce effective mobility

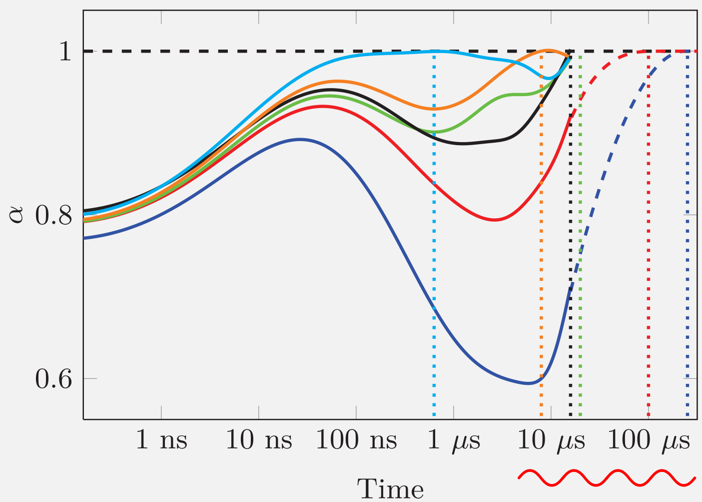


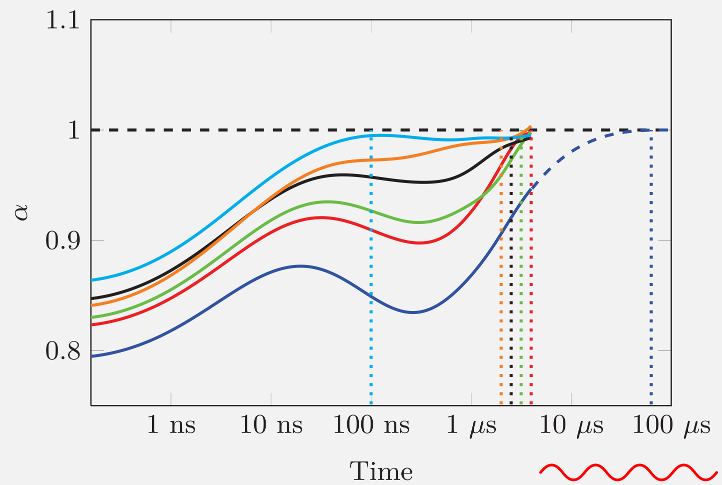


Blue: DLPC. Red: DPPC



Protein crowding effects anomalous lipid diffusion

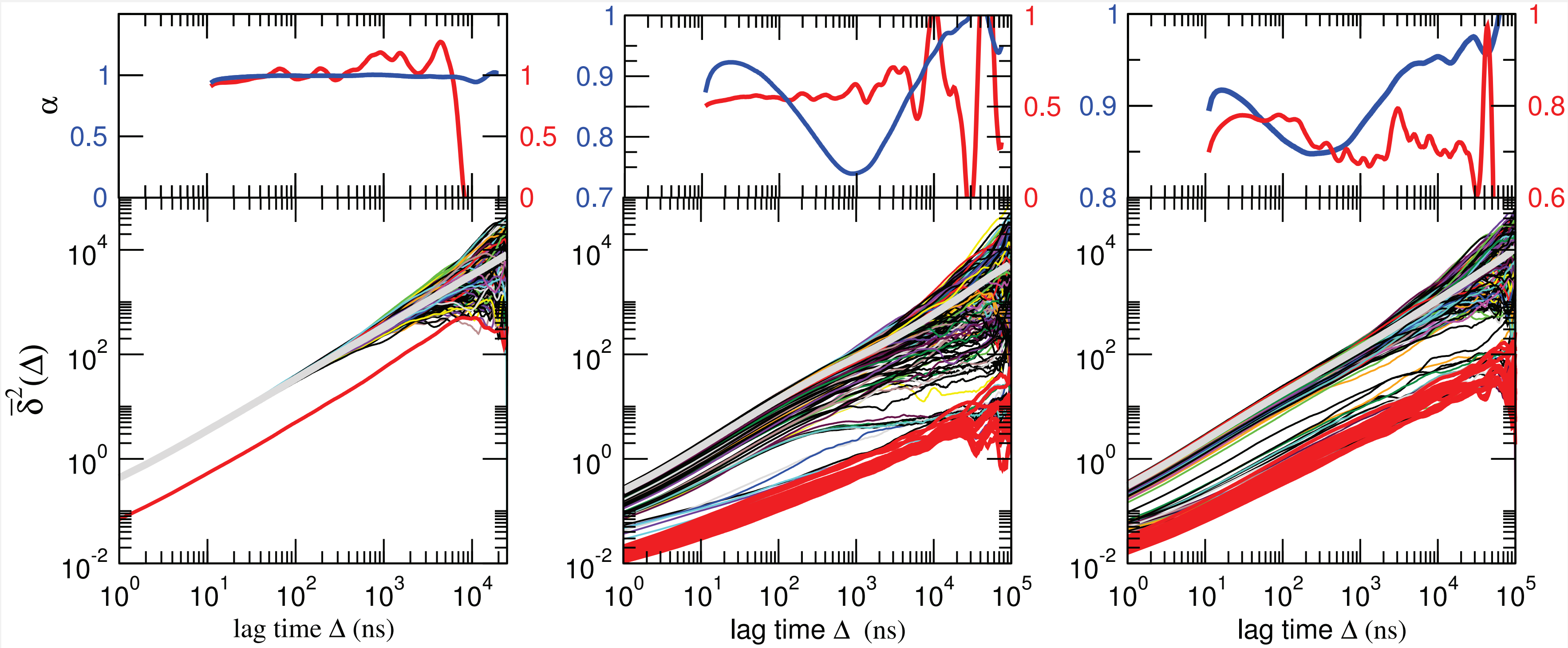




Left: DPPC (protein-aggregating) case. Right: DLPC protein non-aggregating case.

M Javanainen, H Hammaren, L Monticelli, JH Jeon, RM & I Vattulainen, Faraday Disc (2013)

Crowding in membranes increases dynamic heterogeneity



Single NaK channel

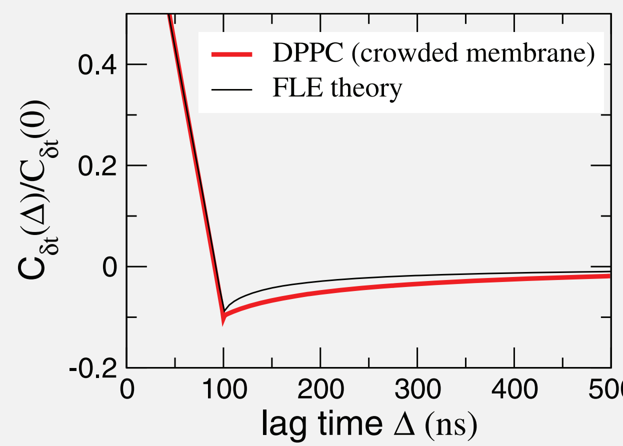
DLPC (non-aggregating)

DPPC (aggregating)

← Blue: lipids. Red: protein(s)

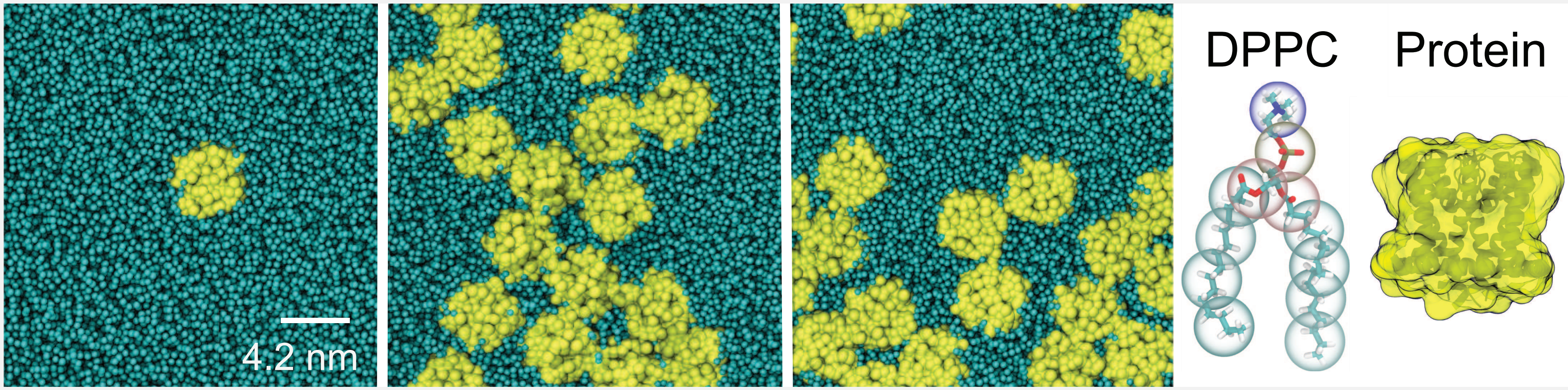
Lipids & proteins behave quite differently

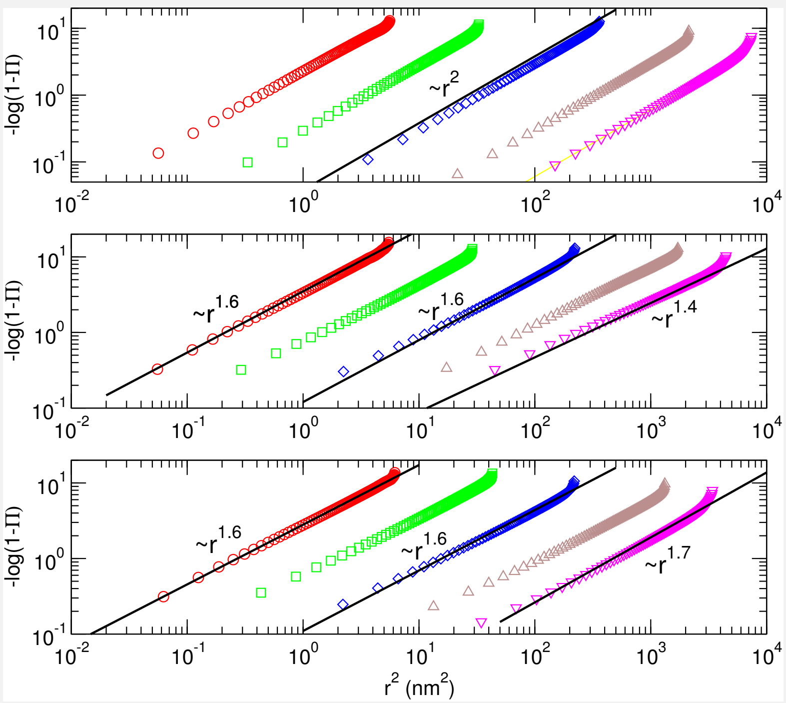
Increment correlation no longer simple FBM ⟶



J-H Jeon, M Javanainen, H Martinez-Seara, RM & I Vattulainen, PRX (2016)

Crowding in membranes: non-Gaussian lipid/protein diffusion

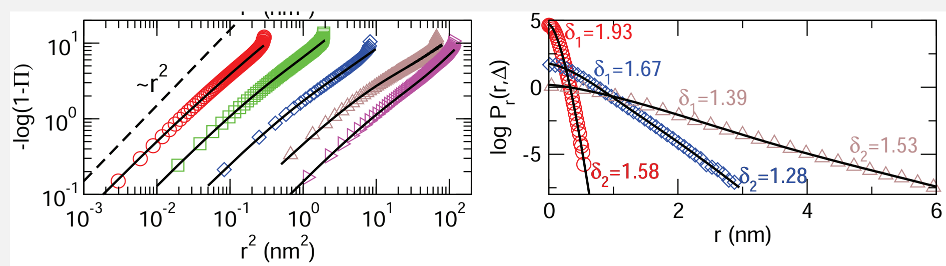




Dilute membrane: $P(r,t)$ Gauss

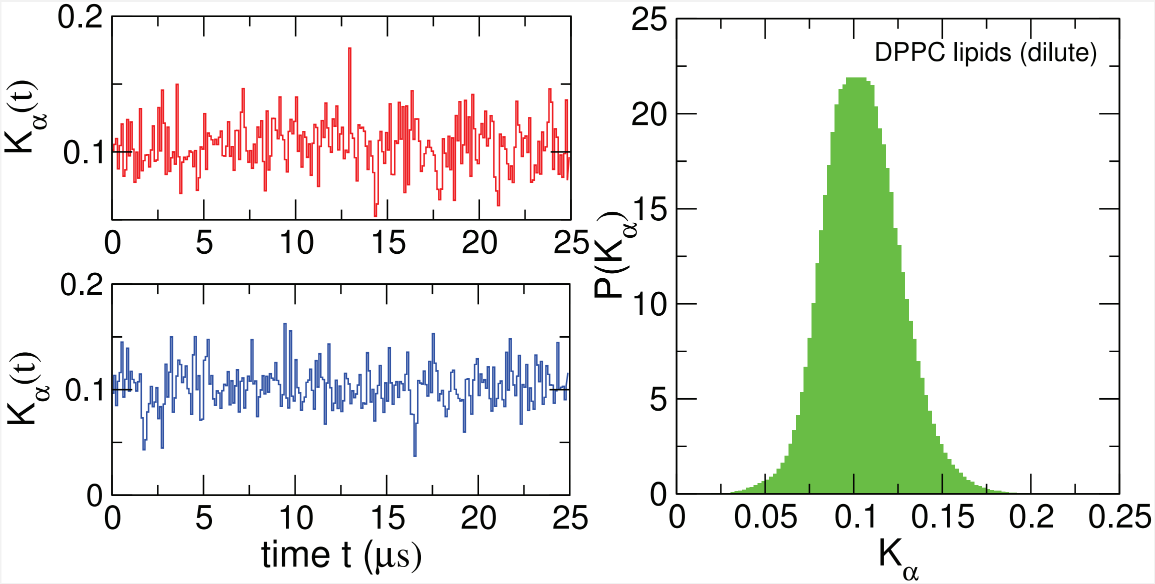
Crowded membrane ($\delta \approx 1.3 \ldots 1.7$):

$$P(r,t) \propto \exp\left(-\left[\frac{r}{ct^{\alpha/2}}\right]^{\delta}\right)$$



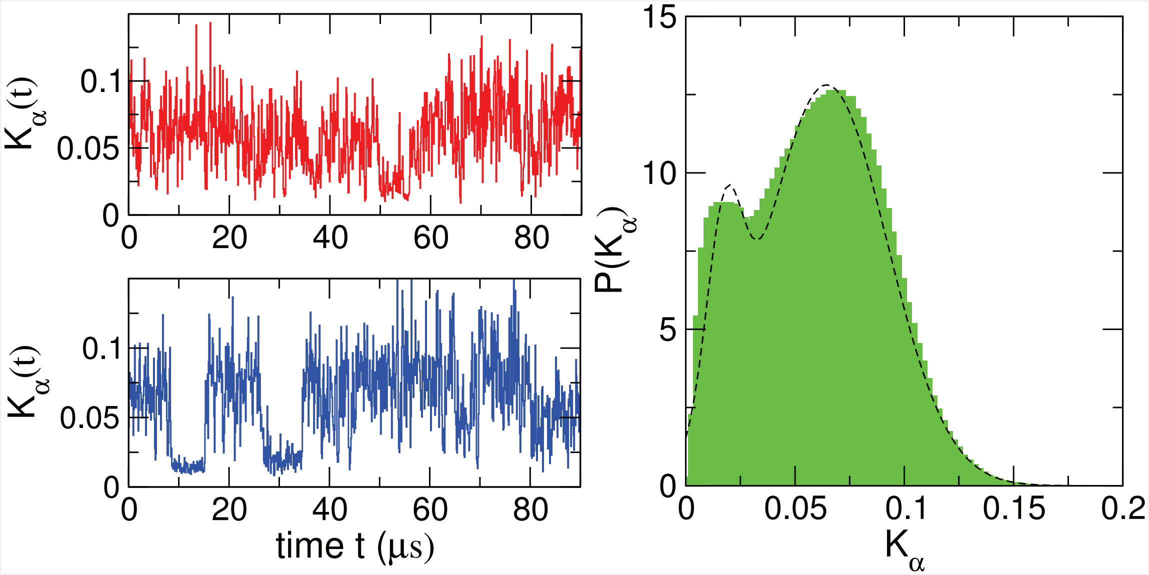
J-H Jeon, M Javanainen, H Martinez-Seara, RM & I Vattulainen, PRX (2016)

Crowding in membranes increases dynamic heterogeneity



Diffusivity(t) for two lipids

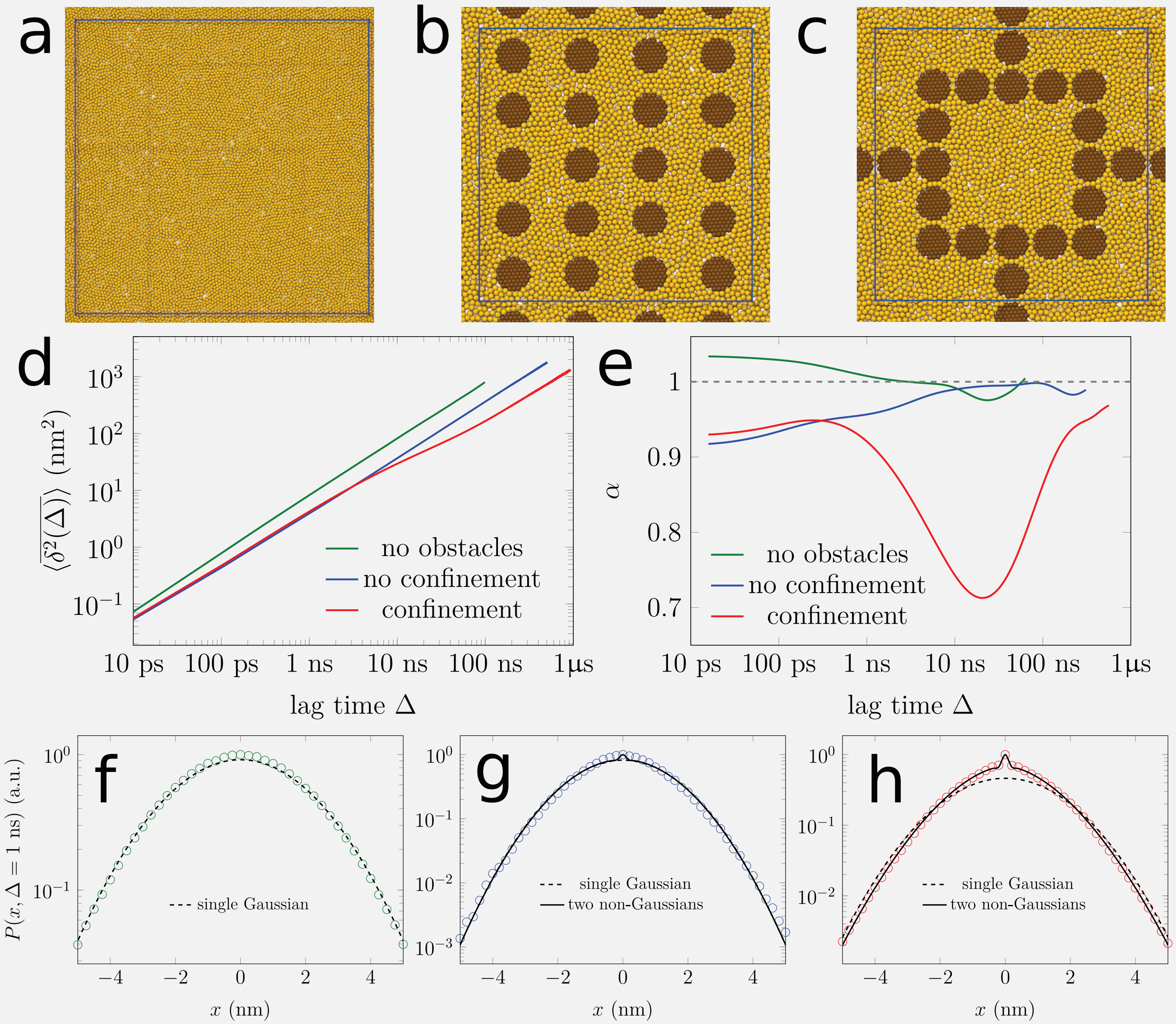
Lipid diffusivity, dilute membrane



Lipid diffusivity, crowded membrane

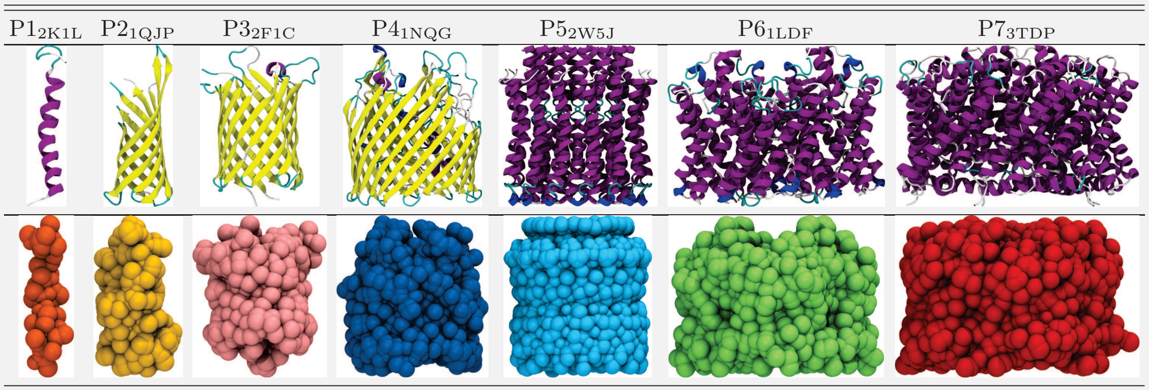
J-H Jeon, M Javanainen, H Seara Monne, RM & I Vattulainen, PRX (2016)

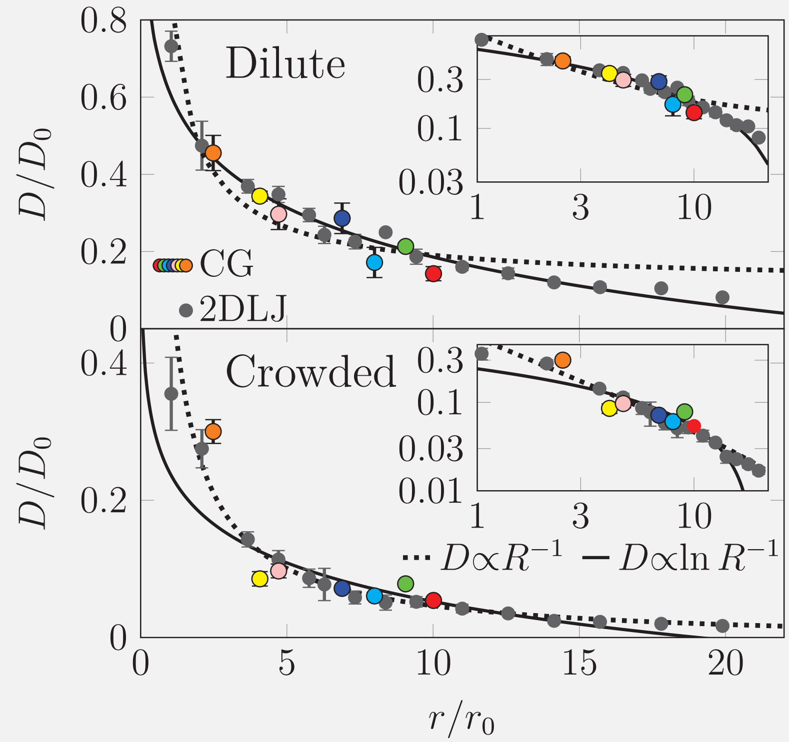
Confinement in argon system shows geometric origin

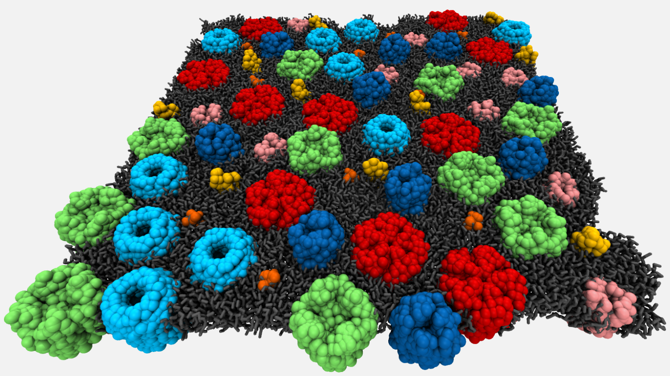


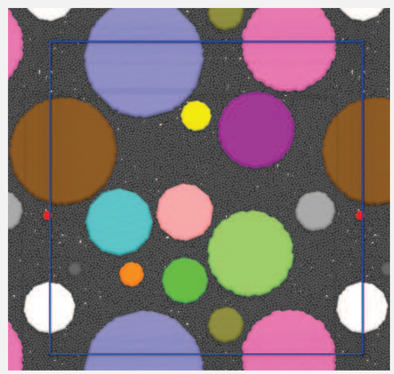
J-H Jeon, M Javanainen, H Seara Monne, RM & I Vattulainen, PRX (2016)

Geometry-induced violation of Saffman-Delbrück relation









Dilute system: Saffman-Delbrück law

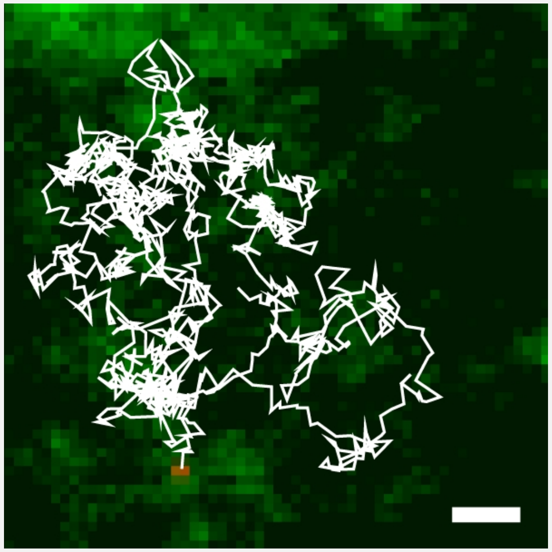
$$D(R) \simeq \log(1/R)$$

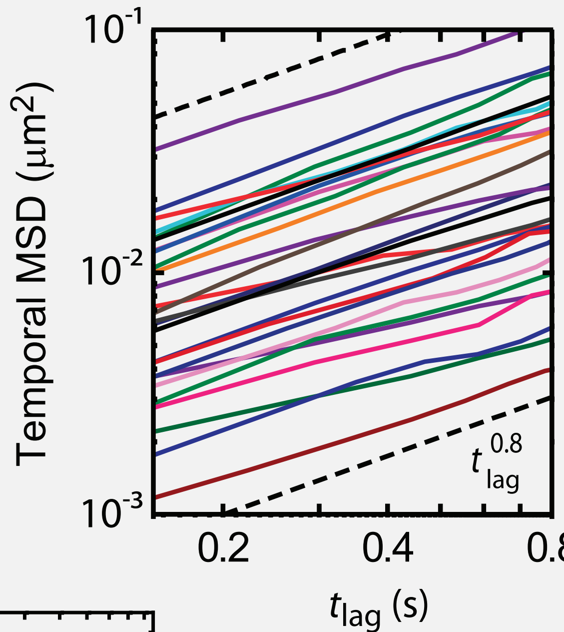
Crowded membrane & 2DLJ discs:

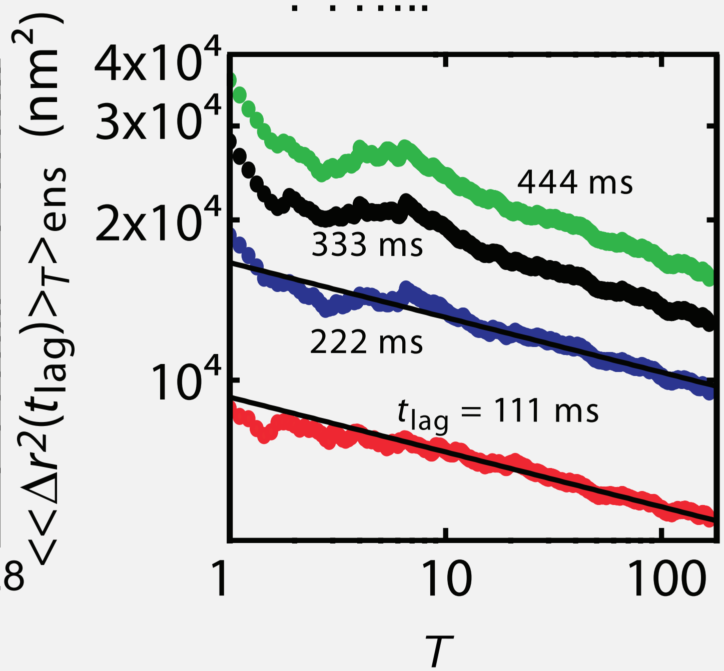
$$D(R) \simeq 1/R$$

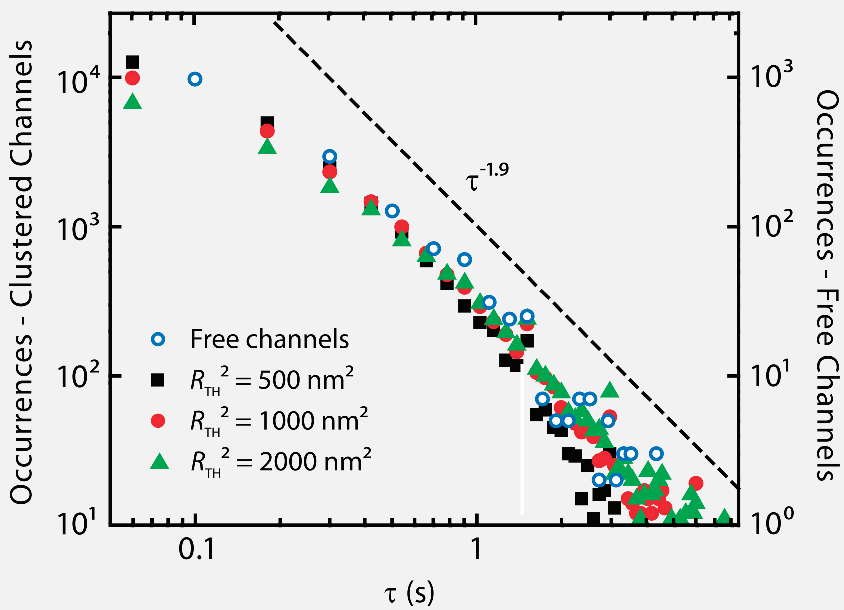
M Javanainen, H Seara Monne, RM & I Vattulainen, JPC Lett (2017)

CTRW-like motion of Ka channels in plasma membrane









$\psi(\tau) \simeq \tau^{-1-\alpha}$ scale free

$\overline{\delta^2(\Delta)}$ apparently random

$\overline{\delta^2(\Delta)} \neq \langle \mathbf{r}^2(\Delta) \rangle$ WEB

AV Weigel, B Simon, MM Tamkun & D Krapf, PNAS (2011)

Time averaged MSD & weak ergodicity breaking (WEB)

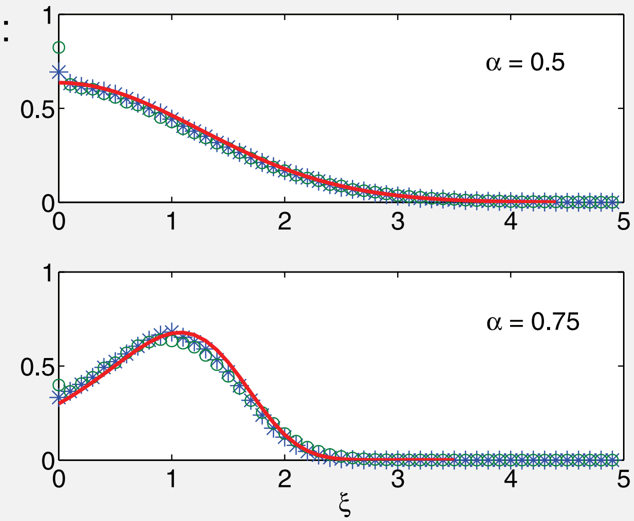
Time averaged MSD $\simeq \Delta$ is pseudo-Brownian and ageing ($\langle x^2(t)\rangle \simeq K_\alpha t^\alpha$):

$$\left\langle \overline{\delta^2(\Delta)} \right\rangle \sim \frac{1}{N}\sum_i^N \overline{\delta_i^2(\Delta)} \sim \frac{2dK_\alpha}{\Gamma(1+\alpha)}\frac{\Delta}{T^{1-\alpha}} \quad \therefore \quad K_\alpha \equiv \frac{\langle \delta \mathbf{r}^2\rangle}{2\tau^\alpha}$$

Amplitude distribution $\overline{\delta^2}$ of trajectories ($\xi \equiv \overline{\delta^2}/\langle\overline{\delta^2}\rangle$):

$$\phi_\alpha(\xi) \sim \frac{\Gamma^{1/\alpha}(1+\alpha)}{\alpha \xi^{1+1/\alpha}} L_\alpha^+\left(\frac{\Gamma^{1/\alpha}(1+\alpha)}{\xi^{1/\alpha}}\right)$$

$$\phi_{1/2}(\xi) = \frac{2}{\pi}\exp\left(-\frac{\xi^2}{\pi}\right); \quad \phi_1(\xi) = \delta(\xi - 1)$$

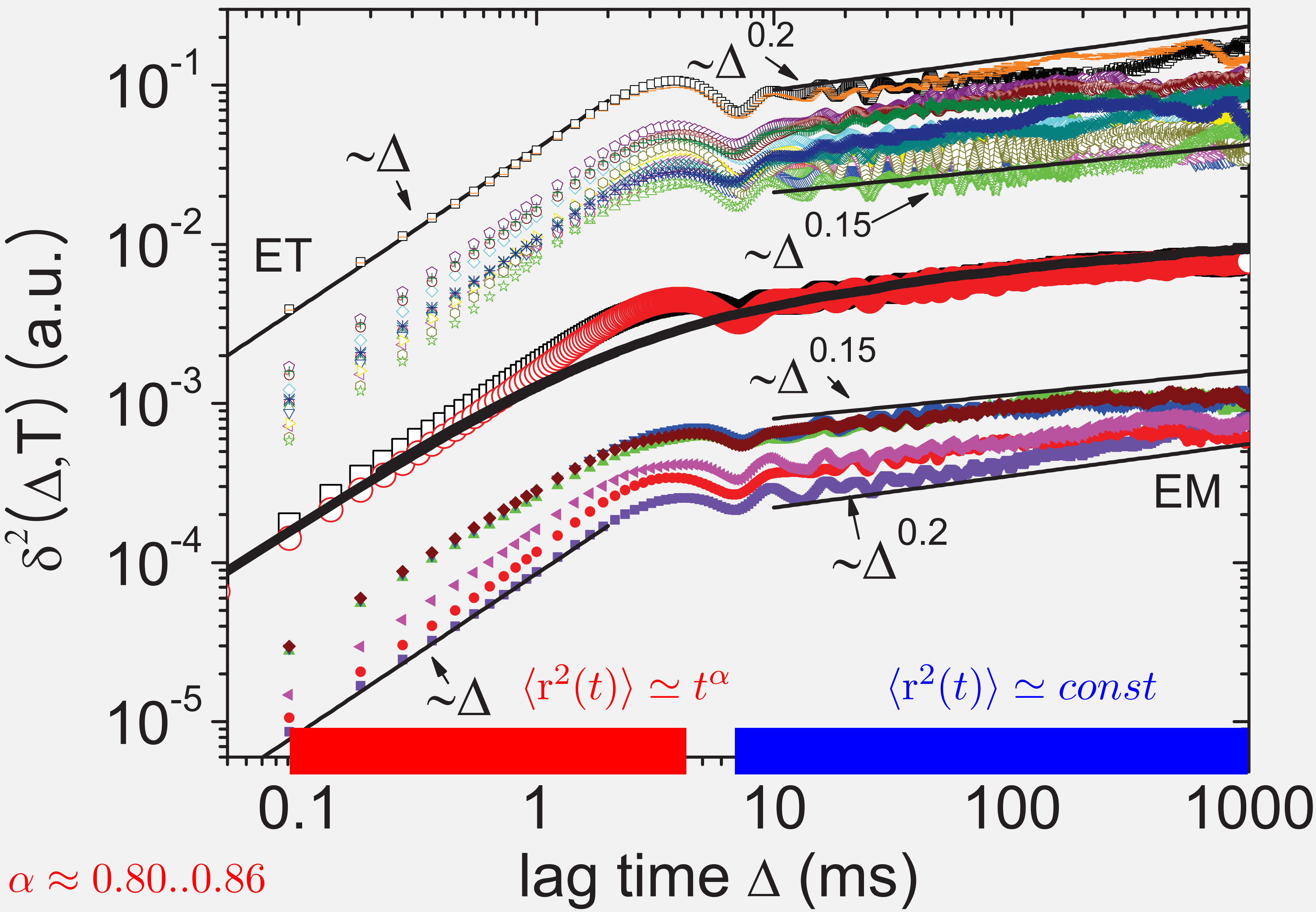


Confinement does not effect a plateau ($\langle x^2(t)\rangle \simeq \mathrm{const}(T)$):

$$\left\langle \overline{\delta^2(\Delta)} \right\rangle \sim \left(\left\langle x^2\right\rangle_B - \langle x\rangle_B^2\right)\frac{2\sin(\pi\alpha)}{(1-\alpha)\alpha\pi}\left(\frac{\Delta}{T}\right)^{1-\alpha}; \quad \frac{1}{(K_\alpha \lambda_1)^{1/\alpha}} \ll \Delta \ll T$$

Y He, S Burov, RM & E Barkai, PRL (2008); S Burov, RM, & E Barkai, PNAS (2010)

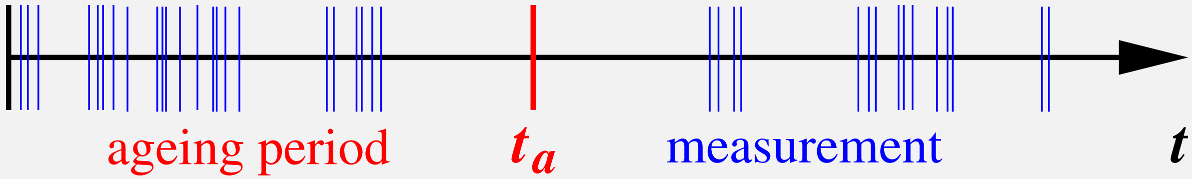
Granule subdiffusion in harmonic optical tweezer potential



$\alpha \approx 0.80..0.86$

J-H Jeon, V Tejedor, S Burov, E Barkai, C Selhuber-Unkel, K Berg-Sørensen, L Oddershede & RM, PRL (2011)

Ageing effects in single trajectory time averages

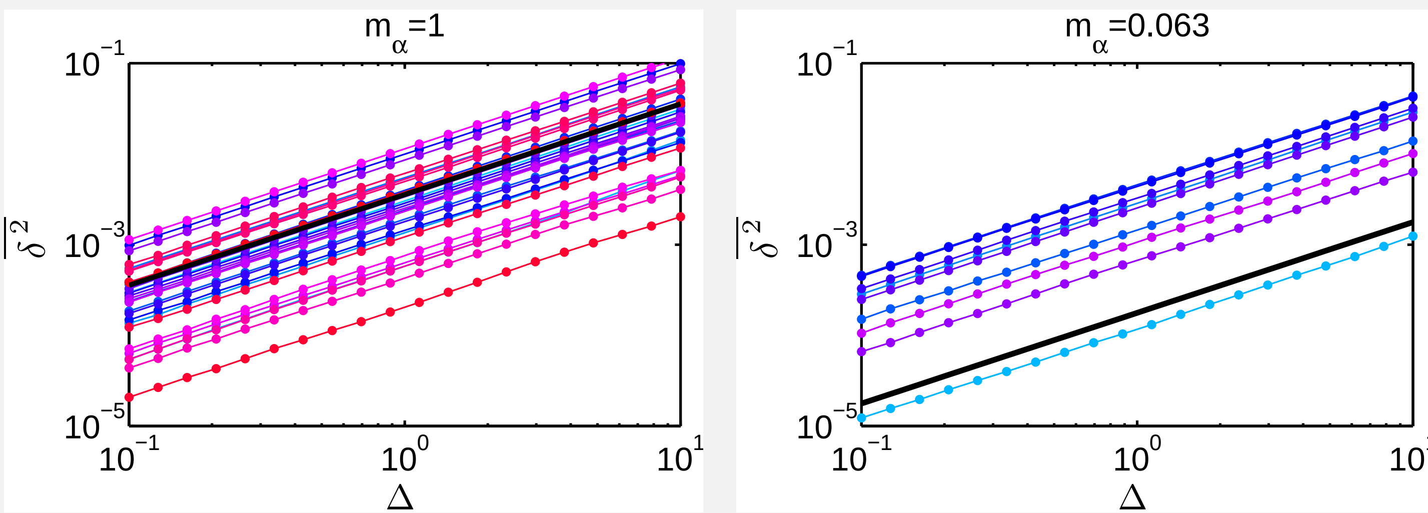


Ageing mean squared displacement ($\Lambda(z) = (1+z)^\alpha - z^\alpha$)

$$\left\langle \overline{\delta^2(\Delta)} \right\rangle_a = \frac{\Lambda_\alpha(t_a/T)}{\Gamma(1+\alpha)} \frac{g(\Delta)}{T^{1-\alpha}} \quad \not\Leftrightarrow \quad \langle x^2(t) \rangle_a \simeq \begin{cases} t^\alpha, & t_a \ll t \\ t_a^{\alpha-1} t, & t_a \gg t \end{cases}$$

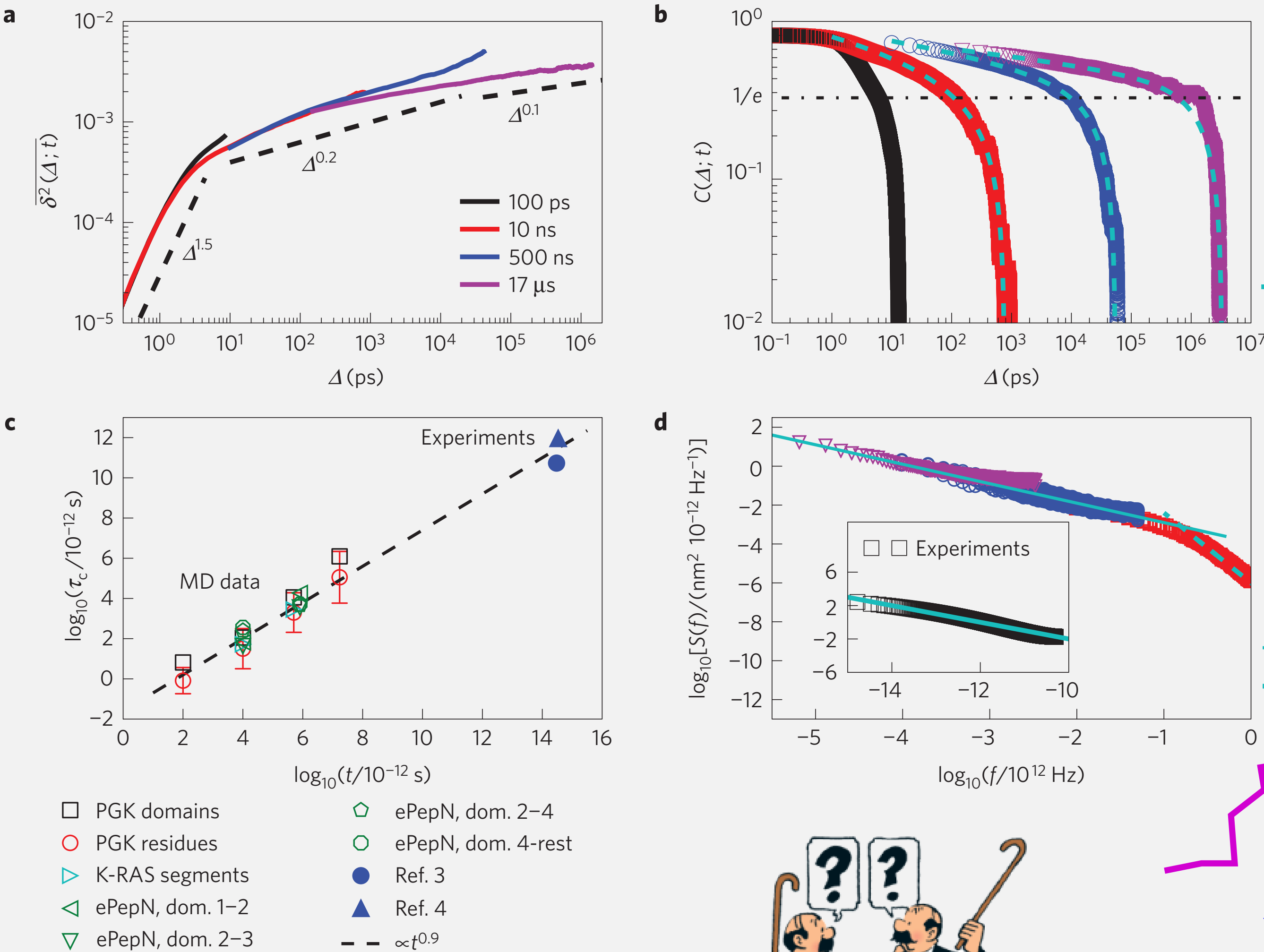
Probability to make at least one step during $[t_a, t_a + T]$: *population splitting*

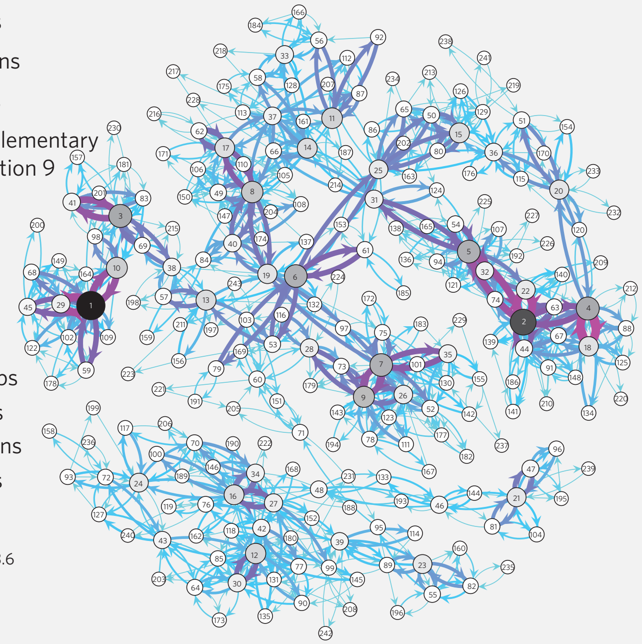
$$m_\alpha(T/t_a) \simeq (T/t_a)^{1-\alpha}, \quad T \ll t_a$$

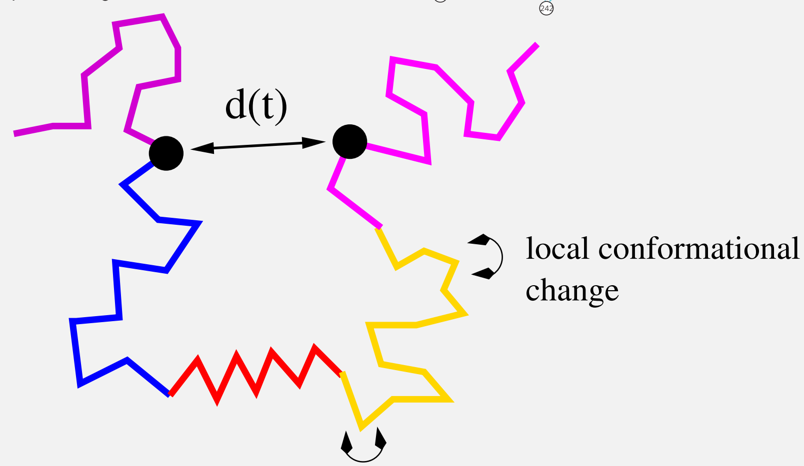


J Schulz, E Barkai & RM, PRL (2013), PRX (2014)

Self-similar internal protein dynamics: 13 decades of ageing

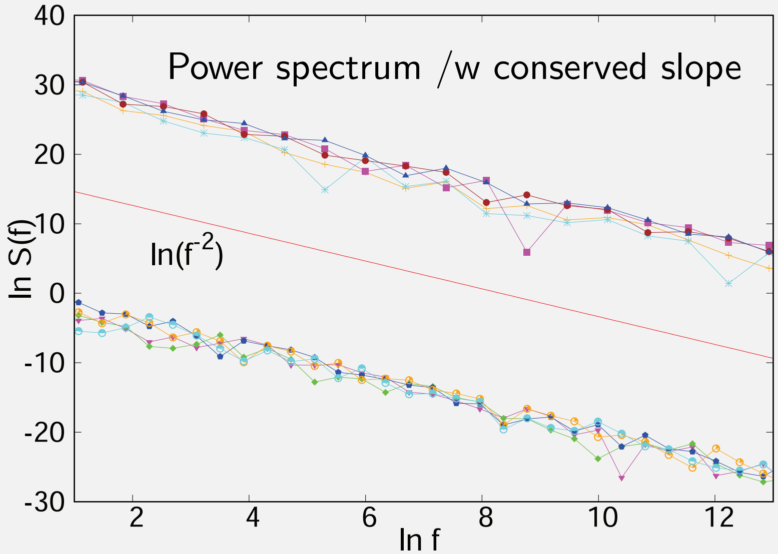


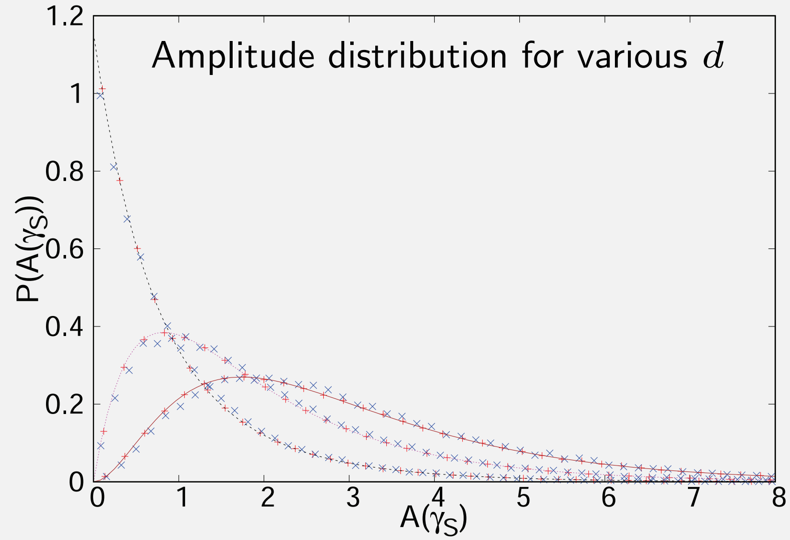




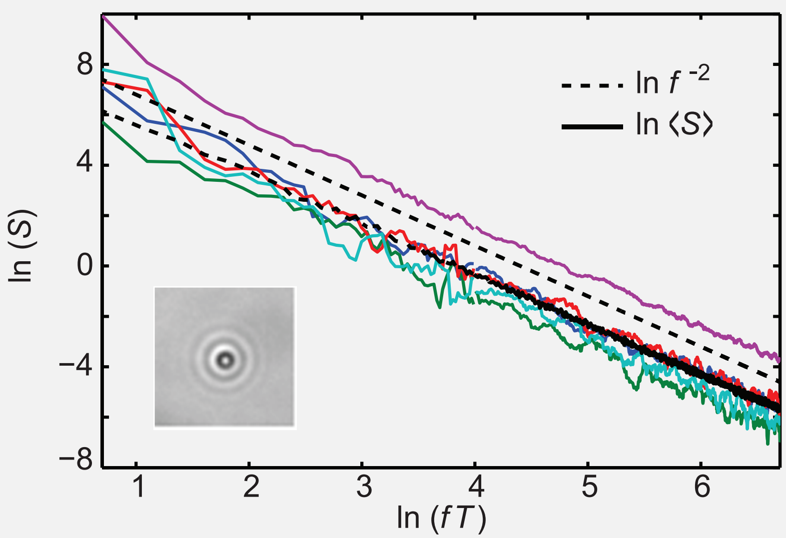
X Hu, L Hong, MD Smith, T Neusius, X Cheng & JC Smith, Nature Phys (2016); N&V RM Nature Phys (2016)

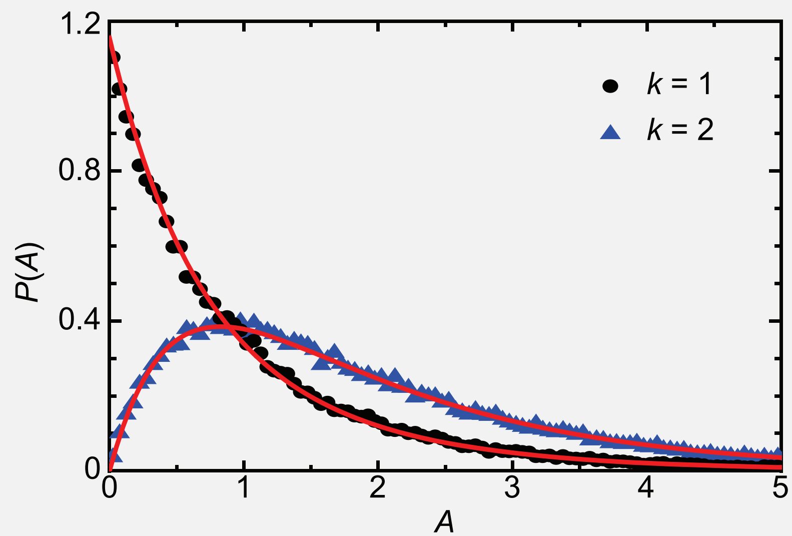
Power spectral density of a single Brownian trajectory





Theory

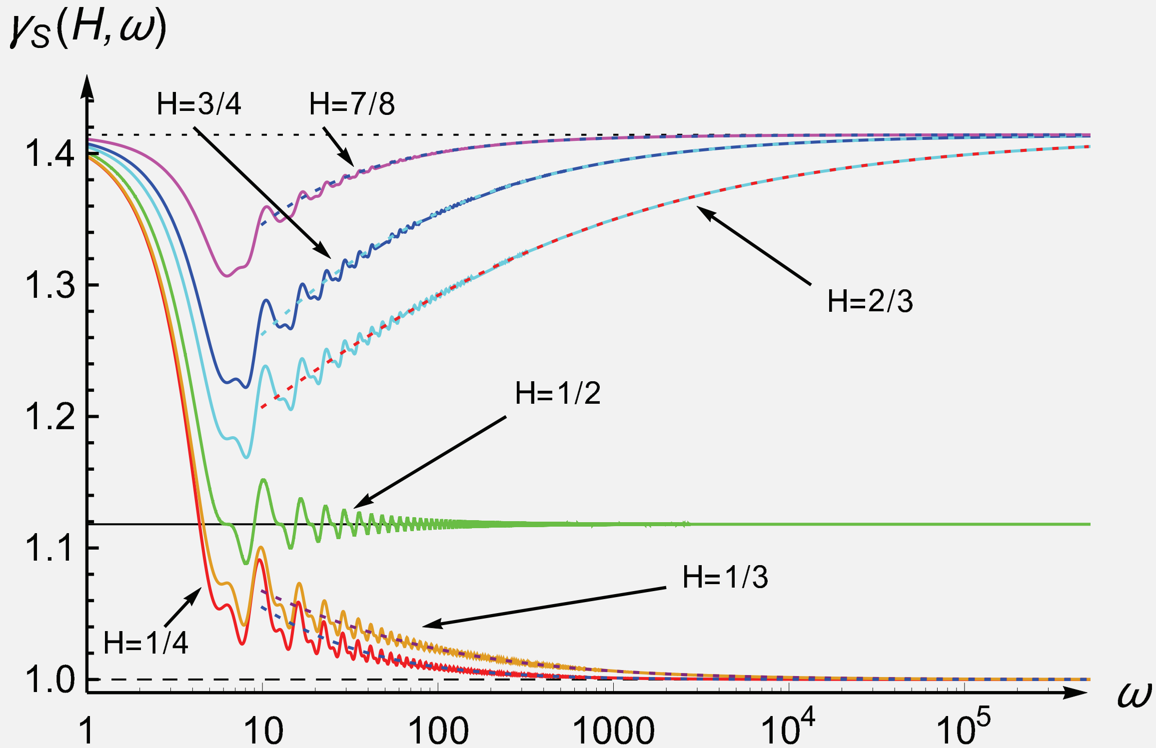




Experiment

D Krapf, E Marinari, RM, G Oshanin, A Squarcini & X Xu, NJP (2018); Perspective: ND Schnellbächer & US Schwarz, NJP (2018)

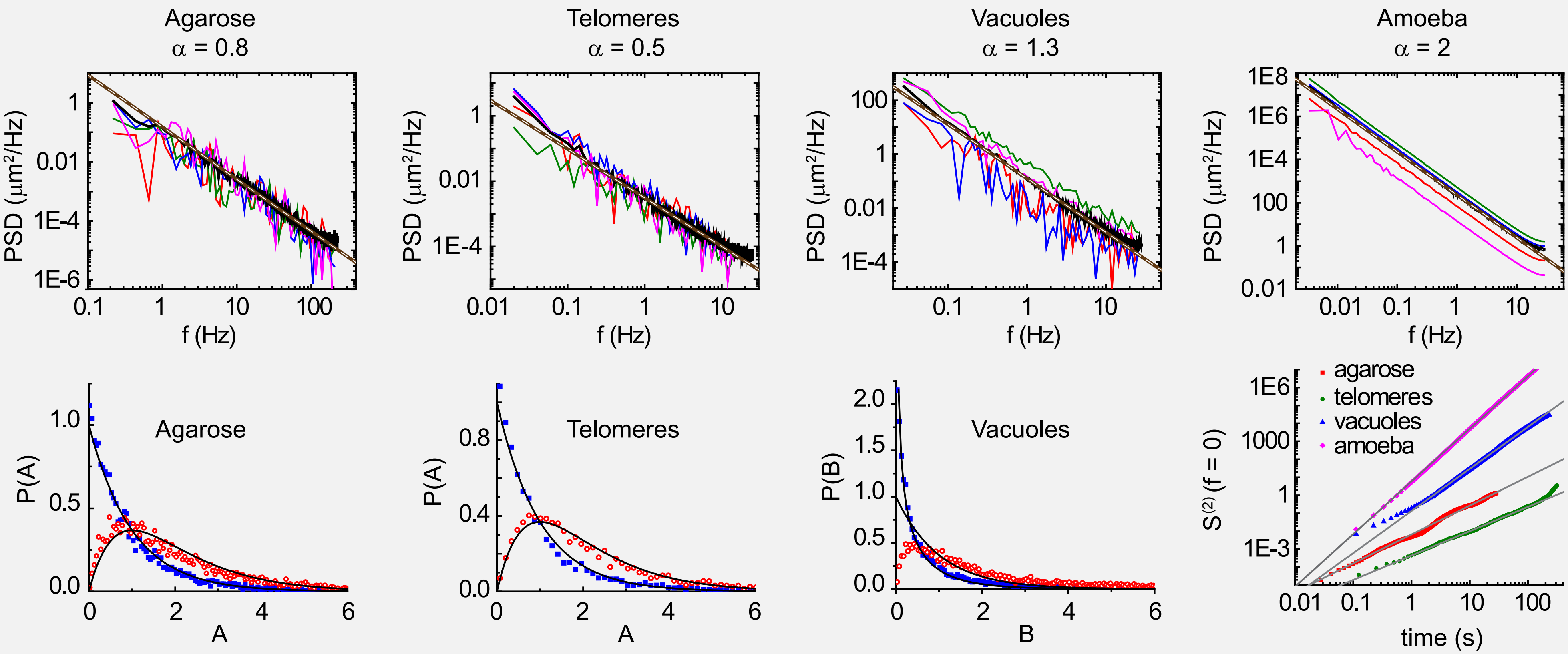
Power spectral density of a single FBM trajectory



$$\gamma = \frac{\left(\langle S_T^2(f)\rangle - \langle S_T(f)\rangle^2\right)^{1/2}}{\langle S_T(f)\rangle}$$

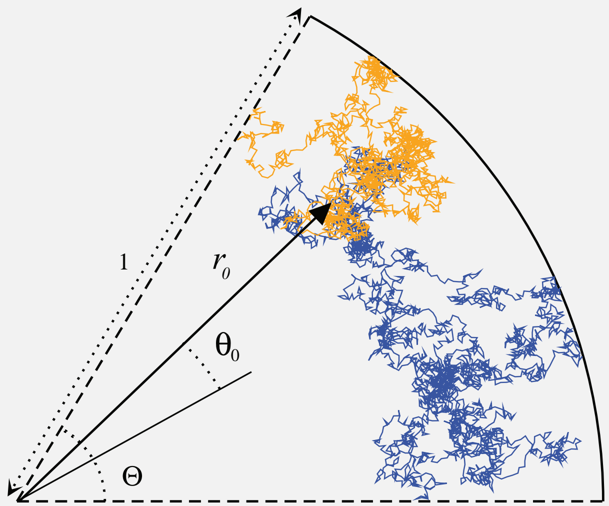
D Krapf, N Lukat, E Marinari, RM, G Oshanin, C Selhuber-Unkel, A Squarcini, L Stadler, M Weiss & X Xu (2018)

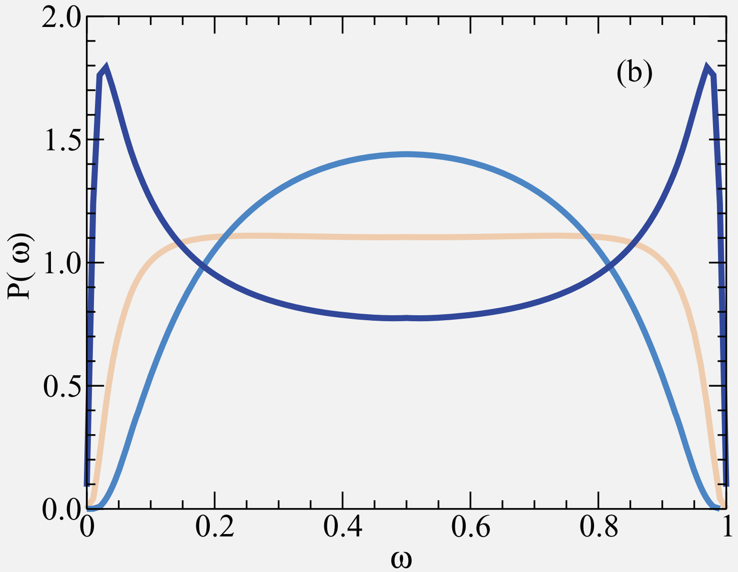
Power spectral density of a single FBM trajectory

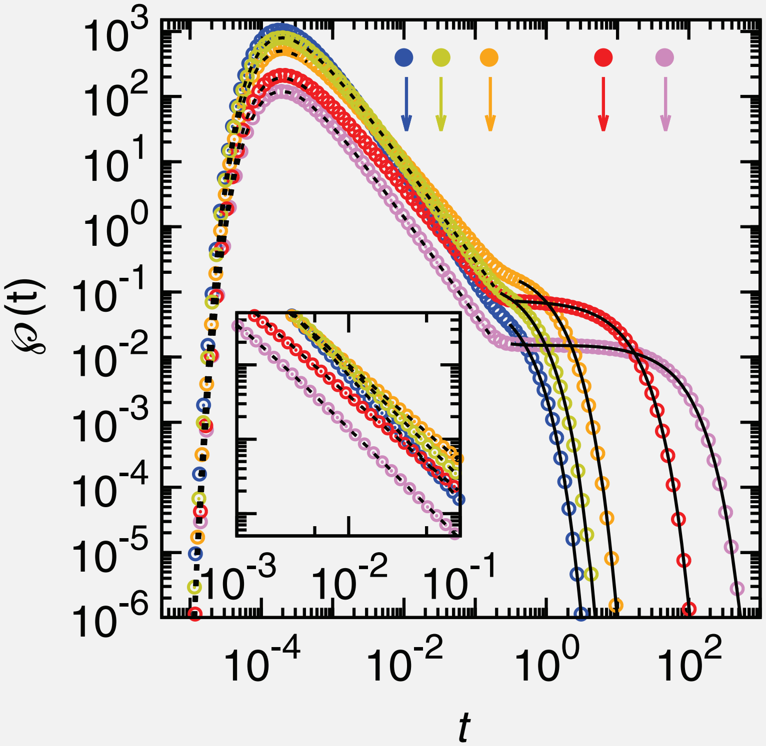


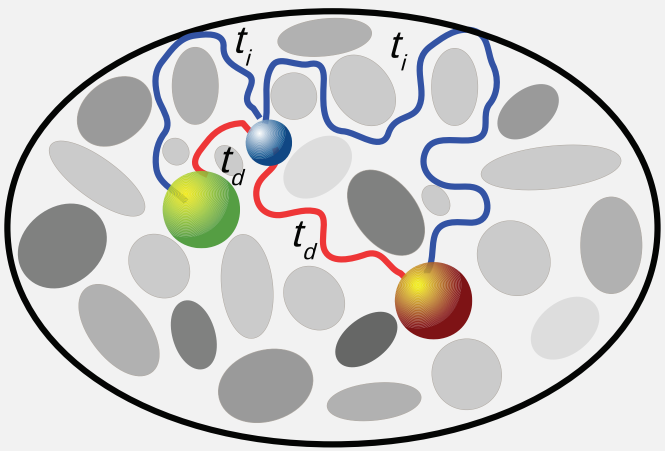
D Krapf, N Lukat, E Marinari, RM, G Oshanin, C Selhuber-Unkel, A Squarcini, L Stadler, M Weiss & X Xu (2018)

First-past-the-post: few-encounter limit & geometry control



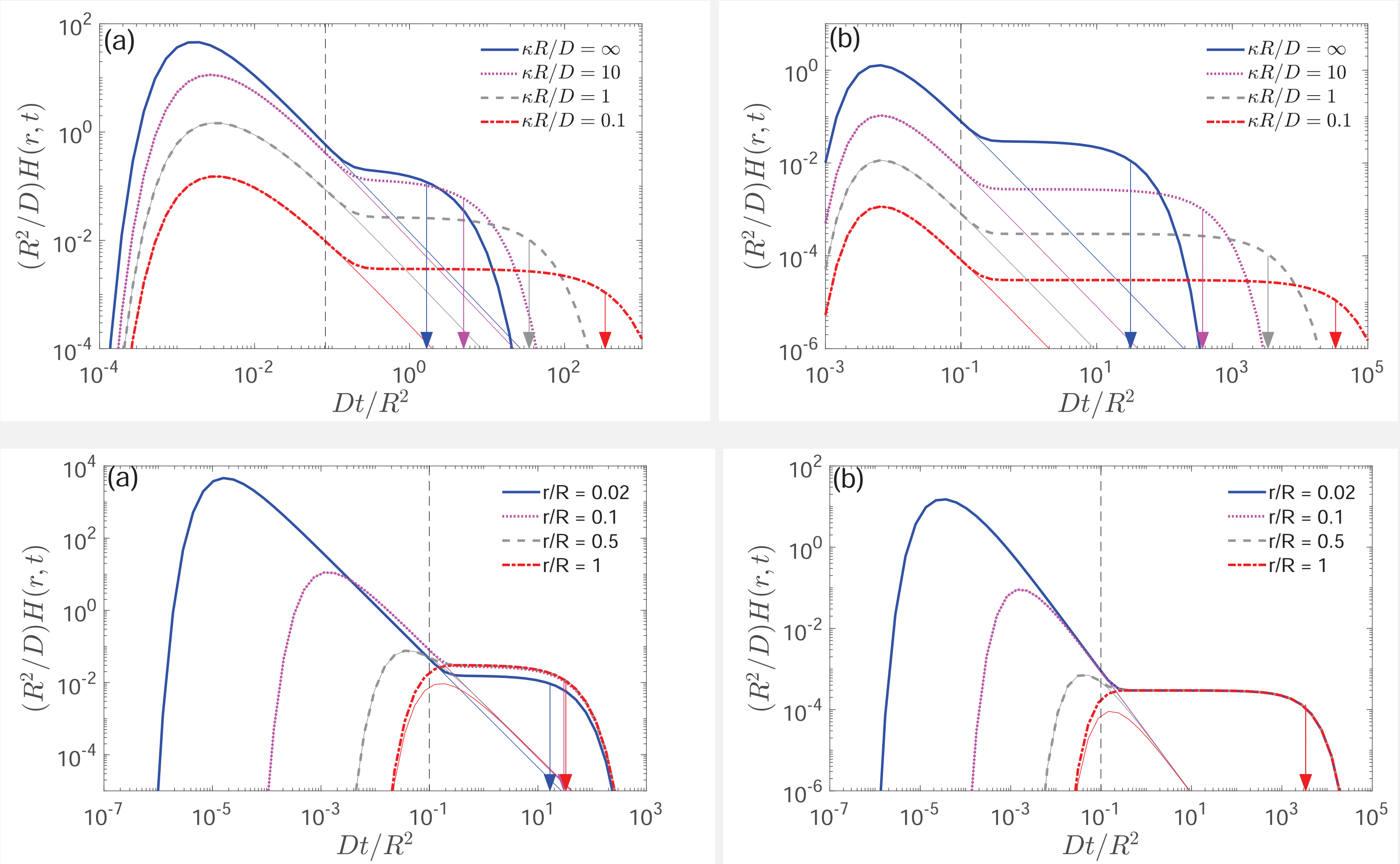






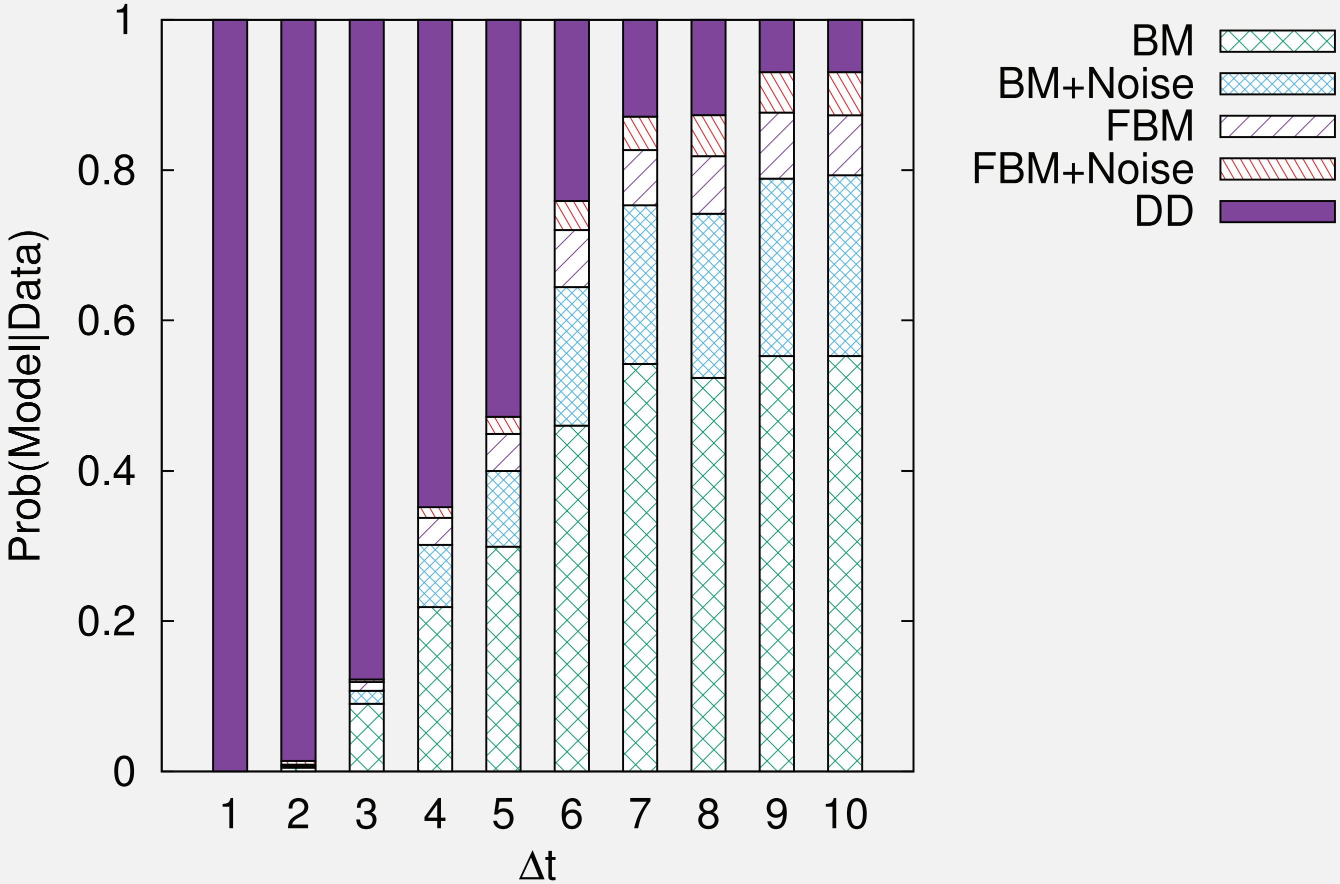
T Mattos, C Mejía-Monasterio, RM & G Oshanin, PRE (2012); A Godec & RM, PRX (2016); Sci Rep (2016)

Geometry control, defocusing, & finite target reactivity



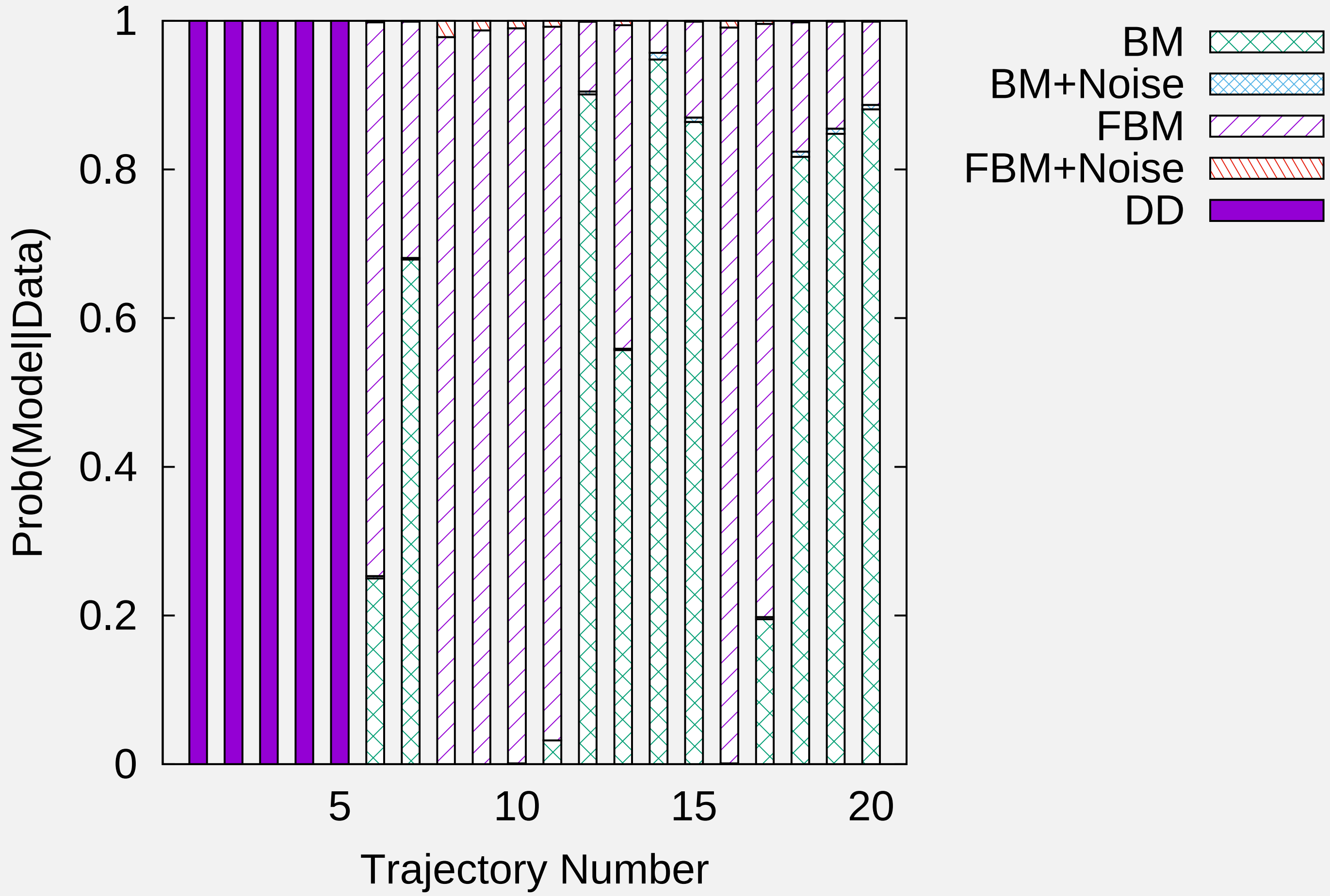
D Grebenkov, RM & G Oshanin, NJP (2017); PCCP (2018); E-print (2018)

Maximum likelihood implementation of diffusing diffusivity



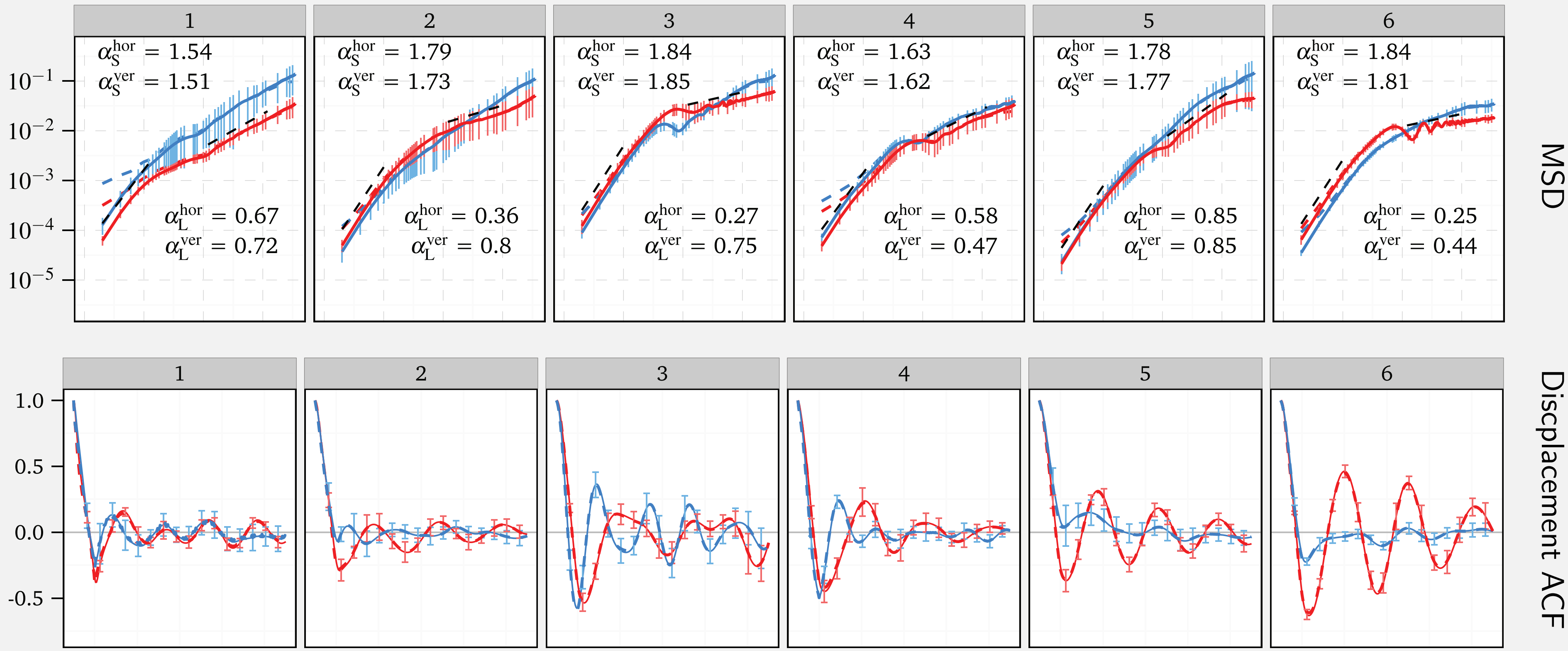
S Thapa, AG Cherstvy & RM (2018); model see AV Chechkin, F Seno, RM & IM Sokolov, PRX (2017)

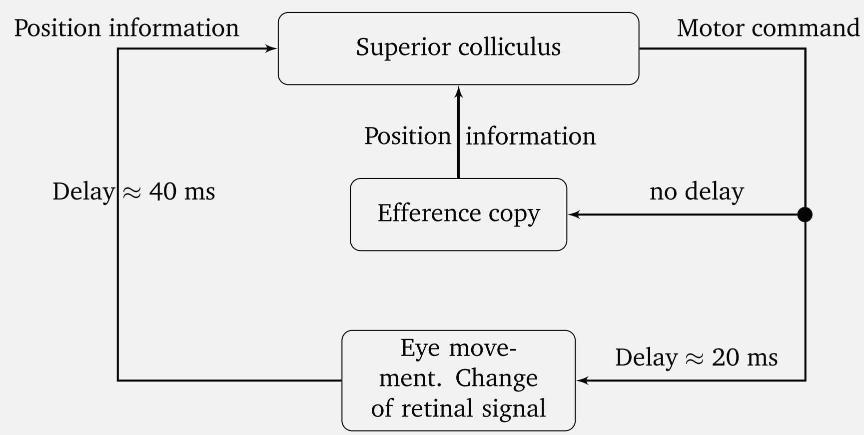
Application to mucin data



S Thapa, AG Cherstvy & RM (2018); model see AV Chechkin, F Seno, RM & IM Sokolov, PRX (2017)

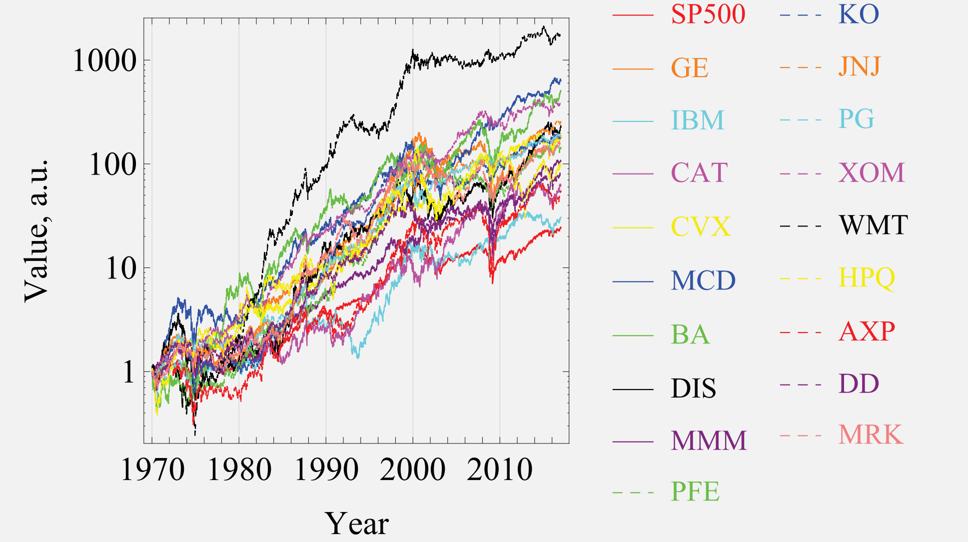
Stochasticity of fixational eye movements

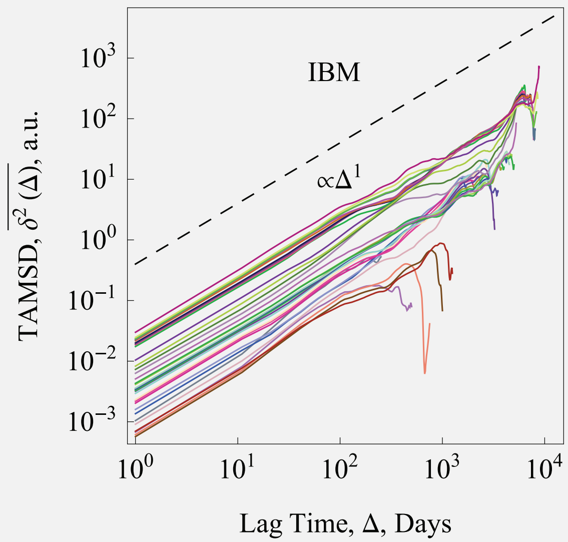


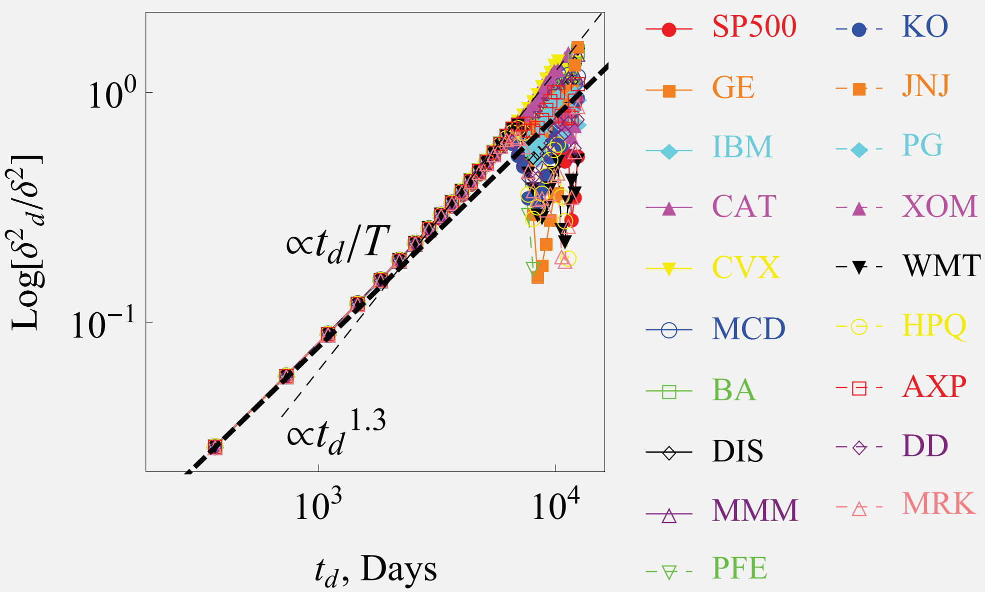


CJJ Herrmann, RM & R Engbert, Sci Rep (2017)

Time averages & ageing in financial market time series







$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t)$$

$$\overline{\delta_d^2(\Delta)} = \frac{\int_{t_d}^{T-\Delta} [X(t+\Delta) - X(t)]^2 \, dt}{T - t_d - \Delta} \sim \frac{\Delta}{T - t_d} X_0^2 \left(e^{\sigma^2 T} - e^{\sigma^2 t_d}\right)$$

$$\log\left[\left\langle \overline{\delta_d^2(\Delta, t_d)} \right\rangle \Big/ \left\langle \overline{\delta^2(\Delta)} \right\rangle\right] \sim t_d/T$$

AG Cherstvy, D Vinod, E Aghion, AV Chechkin & RM, NJP (2017)

Overview articles

- Single particle manipulation & tracking:
  C Nørregaard, RM, CM Ritter, K Berg-Sørensen & LB Oddershede, Chem Rev **117**, 4342 (2017)

- Anomalous diffusion models, WEB & ageing:
  RM, JH Jeon, AG Cherstvy & E Barkai, Phys Chem Chem Phys **16**, 24128 (2014)

- Ageing renewal theory:
  JHP Schulz, E Barkai & RM, Phys Rev X **4**, 011028 (2014)

- Anomalous diffusion in membranes:
  RM, JH Jeon & AG Cherstvy, Biochimica et Biophysica Acta - Biomembranes **1858**, 2451 (2016)

- Polymer translocation:
  V Palyulin, T Ala-Nissila & RM, Soft Matter **10**, 9016 (2014)