Totales Differential.

Grosses Potential: $d\Omega = d(E - TS - \mu N) = dE - TdS - SdT - \mu dN - Nd\mu$

$$= TdS - pdV + \mu dN - TdS - SdT - \mu dN - Nd\mu$$

$$= -SdT - pdV - Nd\mu \rightsquigarrow \Omega = \Omega(T, V, \mu)$$

$$S = -\left(\frac{\partial\Omega}{\partial T}\right)_{V,\mu} ; \quad p = -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\mu} ; \quad N = -\left(\frac{\partial\Omega}{\partial\mu}\right)_{T,V}$$

Gibbspotential (Freie Enthalpie): $dG = d(E + pV - TS) = TdS - pdV + \mu dN + pdV + Vdp - TdS - SdT$

$$= -SdT + Vdp + \mu dN$$

$\rightsquigarrow G = G(T, p, N)$ & $S = -\left(\frac{\partial G}{\partial T}\right)_{p,N} ; \quad V = \left(\frac{\partial G}{\partial p}\right)_{T,N} ; \quad \mu = \left(\frac{\partial G}{\partial N}\right)_{T,p}$

4.2. Teilchenzahlfluktuationen. Äquivalent zu anderen Ensemblen?

Mittlere Teilchenzahl: $\bar{N} = \frac{1}{Z_{gross}} \sum_{N=0}^{\infty} N \exp\left(\frac{\mu N}{k_B T}\right) \int \exp\left(-\frac{H}{k_B T}\right) \frac{d\Gamma}{h^{3N} N!}$

$$\rightsquigarrow \bar{N} = \frac{k_B T}{Z_{gross}} \frac{\partial}{\partial\mu} \sum_{N=0}^{\infty} \exp(\beta\mu N) \int \exp(-\beta H) \frac{d\Gamma}{h^{3N} N!} = \frac{1}{\beta Z_{gross}} \frac{\partial}{\partial\mu} Z_{gross} = \frac{1}{\beta} \frac{\partial \log Z_{gross}}{\partial\mu}$$

$\rightsquigarrow \bar{N} = -\frac{\partial\Omega}{\partial\mu}$ wie aus dem totalen Differential

$$\overline{N^2} = \frac{1}{Z_{gross}} \sum_{N=0}^{\infty} N^2 \exp(\beta\mu N) \int \exp(-\beta H) \frac{d\Gamma}{h^{3N} N!} = \frac{1}{\beta^2 Z_{gross}} \frac{\partial^2 Z_{gross}}{\partial\mu^2}$$

$$\Rightarrow \overline{\delta N^2} = \overline{N^2} - \bar{N}^2 = \frac{1}{\beta^2 Z_{gross}} \frac{\partial^2 Z_{gross}}{\partial\mu^2} - \frac{1}{\beta^2} \left(\frac{\partial \log Z_{gross}}{\partial\mu}\right)^2 = \frac{1}{\beta^2} \frac{\partial^2 \log Z_{gross}}{\partial\mu^2}$$

$$\rightsquigarrow \overline{\delta N^2} = -\frac{1}{\beta} \frac{\partial^2\Omega}{\partial\mu^2}$$

$\rightsquigarrow$ relat. Schwankung der Teilchenzahl: $\frac{\sqrt{\overline{\delta N^2}}}{\bar{N}} = \sqrt{\left|\frac{1}{\beta} \frac{\partial^2\Omega}{\partial\mu^2}\right| \left(\frac{\partial\Omega}{\partial\mu}\right)^{-2}}$

Extensivität von Ω: $\Omega(T, \lambda V, \mu) = \lambda\,\Omega(T, V, \mu) \Rightarrow$ mit $\lambda = \frac{1}{V}$: $\Omega(T, V, \mu) = V\Omega(T, 1, \mu)$