Editors' Suggestion

Fastest First-Passage Time for Multiple Searchers with Finite Speed

Denis S. Grebenkov,[1] Ralf Metzler,[2] and Gleb Oshanin[3]

[1]Laboratoire de Physique de la Matière Condensée (UMR 7643), CNRS—Ecole Polytechnique, IP Paris, 91120 Palaiseau, France
[2]Institute of Physics and Astronomy, University of Potsdam, 14476 Potsdam-Golm, Germany
[3]Sorbonne Université, CNRS, Laboratoire de Physique Théorique de la Matière Condensée (UMR CNRS 7600), 4 Place Jussieu, 75252 Paris Cedex 05, France

(Received 17 February 2026; accepted 3 August 2026; published 15 September 2026)

Deploying N Brownian searchers reduces the mean search time for an immobile target merely by a factor $1/\ln N$. We show that this conclusion is highly sensitive to the short-time dynamics of individual searchers. Based on the telegrapher's equation we demonstrate that the classical logarithmic scaling is only an intermediate asymptotic and ultimately crosses over to a much faster exponential decay with N. For anomalous diffusion with finite speed, we also show that superdiffusion leads to faster search than subdiffusion, as intuitively expected. Our findings provide a clear physical picture of redundancy and minimal search times in collective search relevant for many biological and physical systems.

DOI: 10.1103/7q39-h9dv

In many biological contexts, diverse agents must efficiently locate distant targets—such as transcription factors binding to specific operators on the cellular DNA, immune cells detecting pathogens, or signaling molecules reaching their conjugate receptors [1–3]. These processes are inherently stochastic and often proceed in crowded dynamic environments such as cellular cytoplasm. Although the first-passage statistics for individual searchers are quite well understood [4–6], biological systems rarely rely on a single agent; instead, multiple searchers are deployed in parallel to speed up target detection [2,7,8].

When N searchers begin simultaneously, target detection is determined not by a single agent's shortest arrival time τ, but competitively by the earliest arrival time among all searchers. The problem thus shifts from individual first-passage times τ_k, $k = 1, 2, \ldots, N$, to the statistic of the extremal random variable $\mathcal{T}_N = \min\{\tau_1, \ldots, \tau_N\}$—the fastest first-passage time (fFPT). Order statistics therefore play a central role: the key aspects are the fFPT distribution and the scaling of its moments with N, characterizing the efficiency of such a multiagent search. The analysis of such multiple searcher dynamics reveals how redundancy improves both the reliability and speed of biological search, offering insight into how living systems may optimize their performance.

Most existing analyses of multiagent search processes assume that all agents start from the same point simultaneously and move independently as Brownian walkers with the same diffusion coefficient D [9–17]. Their positions follow Langevin dynamics with Gaussian white noise, and the position probability density function (PDF) of each searcher evolves according to the diffusion equation from $t = 0$, a framework that accurately captures the long-time behavior of many stochastic processes in nature. Within this setting, the *mean* fFPT to a target at a distance x_0 follows the inverse-logarithmic law [9–16]

$$\overline{\mathcal{T}_N} \simeq \frac{x_0^2}{4D\ln(N)} \quad (N \to \infty). \tag{1}$$

Here, the bar denotes averaging with respect to individual trajectory realizations of all searchers, and the symbol "$\simeq$" signifies that we consider solely the leading-order behavior in the limit $N \to \infty$. This result was first derived for one-dimensional continuous-space systems, but later shown to hold in bounded domains of any dimension [18,19], because it is dominated by so-called "direct" trajectories [20,21] that go straight to the target, rendering the actual embedding spatial dimension irrelevant.

The asymptotic form (1) implies that the mean fFPT decreases only as $1/\ln N$ when additional searchers are recruited. Given that the associated biological cost generally grows linearly with N, this reduction is surprisingly modest, rendering the classical redundancy strategy highly inefficient. Moreover, in the limit $N \to \infty$, the fastest searcher would reach the distant target arbitrarily quickly—effectively instantaneously. In addition, if the agents undergo a subdiffusive motion with the mean-squared displacement (MSD) $\overline{x^2(t)} \propto t^\alpha$ $(0 < \alpha < 1)$ and a position PDF obeying a fractional diffusion equation [22], it was shown that

$$\overline{\mathcal{T}_N} \simeq \frac{t_\alpha}{[\ln(N)]^{\frac{2}{\alpha}-1}} \quad (N \to \infty), \tag{2}$$

where $t_\alpha = \alpha(2-\alpha)^{[(2/\alpha)-1]}[x_0^2/(4D_\alpha)]^{(1/\alpha)}$ is a characteristic timescale and D_α is the generalized diffusivity [23].

 © 2026 American Physical Society

This result indicates that for subdiffusive dynamics, $\overline{\mathcal{T}_N}$ also vanishes as $N \to \infty$. Strikingly, Eq. (2) implies that $\overline{\mathcal{T}_N} \to 0$ when $\alpha \to 0$, suggesting that *slower* diffusion *enhances* the speed of the fastest arrival, as highlighted in the title of [23]. Although mathematically rigorous, the results (1) and (2) are clearly unrealistic for very large N—even the fastest searcher needs a finite time to reach a target a finite distance away, thus underscoring the need for a more refined analysis that yields more plausible behavior. This issue was recently addressed in [24], where an exact large-N form of the fFPT was derived for a class of finite-speed stochastic processes in dimensions 1, 2, and 3.

Here, we revisit this fundamental problem from a different perspective. We reveal that the dependence of the mean fFPT on N is extremely sensitive to the short-time dynamics of individual searchers. The logarithmic scalings in Eqs. (1) and (2) ultimately stem from an artifact of Gaussian white-noise models, which allow a particle to cover arbitrarily large distances in arbitrarily short times. For sufficiently large N, these rare but unphysical trajectories dominate the statistics of $\mathcal{T}_N$, leading to an overestimation of survival probabilities and, consequently, to the asymptotic behaviors predicted by Eqs. (1) and (2). Note that this problem is also well known in heat transport, where it is circumvented by replacing the parabolic diffusion equation by a hyperbolic Cattaneo equation with finite propagation speed [25,26].

To understand this effect, we consider an analytically tractable model of *finite-speed* searchers in one dimension. They start from the same point $x_0 > 0$ at time $t = 0$ and move independently from each other, such that their positions $x_k(t)$ follow the Langevin dynamics

$$\dot{x}_k(t) = \eta_k(t), \qquad x_k(0) = x_0 \quad (k = 1, \ldots, N), \tag{3}$$

where $\eta_k(t)$ are statistically independent *symmetric* dichotomous noises alternating between the values $\pm v$. The switching events occur after random, exponentially distributed time intervals with the rate λ. The mean and autocovariance function of the noises read [27,28] (see also Refs. [29,30] and references therein)

$$\overline{\eta_k(t)} = 0, \qquad \overline{\eta_k(t)\eta_{k'}(t')} = \delta_{k,k'} 2\lambda D e^{-2\lambda|t-t'|}, \tag{4}$$

in terms of the Kronecker-delta $\delta_{k,k'}$ and the long-time diffusion coefficient $D = v^2/(2\lambda)$. Here, $1/(2\lambda)$ is a finite correlation time of the noises. The MSD of $x_k(t)$ scales linearly with time in the long-time limit. Based on this model we focus on the statistics of the first-passage time τ_k of the kth searcher to the origin (a target), and on the above-defined extremal random variable $\mathcal{T}_N$, the fFPT.

This model provides a more realistic framework to capture the short-time dynamics relevant to multiagent search. (a) Dichotomous-noise dynamics generates trajectories with a finite velocity—a searcher cannot travel an arbitrarily large distance in an arbitrarily short time. This removes the pathology of Brownian motion at short times, whose propagator has infinite support at any $t > 0$. (b) Dichotomous noise is a simple solvable model of run-and-tumble dynamics, relevant to bacteria such as *E. coli*, active colloids, and other self-propelled agents that alternate between persistent motion and random reorientation events [31–33]. (c) Symmetric dichotomous noise with alternating velocities encodes strong antipersistence, common in crowded environments, since each forward step is followed by a backward one (see, e.g., Ref. [34]). Last, we note that although we focus on one dimension, the results will remain qualitatively unchanged in higher dimensions. In fact, fastest first-passage events are dominated by the direct trajectories that follow the shortest route to the target, making the leading behavior effectively one-dimensional even in higher-dimensional systems (see, e.g., Refs. [18–20]).

Exact mean fFPT—The first-passage-time PDF for a particle driven by a symmetric dichotomous noise is known exactly [35–38]; for completeness, a derivation is provided in Sec. A of Supplemental Material (SM) [39]. Using this result, we get (see Supplemental Material, Sec. B1 [39]) an exact expression for the mean fFPT $\overline{\mathcal{T}_N}$, which is valid for arbitrary $N \geq 3$ (i.e., N is not necessarily large),

$$\overline{\mathcal{T}_N} = t_{\min}(1 + B_{N,\xi}), \tag{5}$$

where the ballistic travel time $t_{\min} = x_0/v$ is the natural lower bound for $\overline{\mathcal{T}_N}$, and $\xi = \lambda x_0/v$ is the main dimensionless parameter that controls the search efficiency. The dimensionless function $B_{N,\xi} > 0$ measures the excess due to dichotomous dynamics,

$$B_{N,\xi} = \int_1^\infty [f_\xi(y)]^N dy, \tag{6a}$$

$$f_\xi(y) = \xi \int_{\operatorname{arccosh}(y)}^\infty e^{-\xi \cosh z} I_1(\xi \sinh z) dz, \tag{6b}$$

where $I_1(z)$ is the modified Bessel function of the first kind. Note that $B_{N,\xi}$ is a monotonically *increasing* function of ξ and a monotonically *decreasing* function of N, as intuitively expected. In Fig. 1 we plot $B_{N,\xi}$ obtained from numerical evaluation of the integrals in Eqs. (6a) and (6b), which illustrates its overall behavior and clearly highlights these features.

Before we discuss the behavior of expression (5), we consider an "order of magnitude" estimate of ξ for several physical systems. For instance, the diffusive dynamics of a small protein or of an ion, say K^+, in water, has a noise correlation time of the order of picoseconds, $(2\lambda)^{-1} \sim 10^{-12}$ s. The diffusion coefficients of a small protein or of an ion are of the order of 10^2 $\mu m^2/s$ [40]

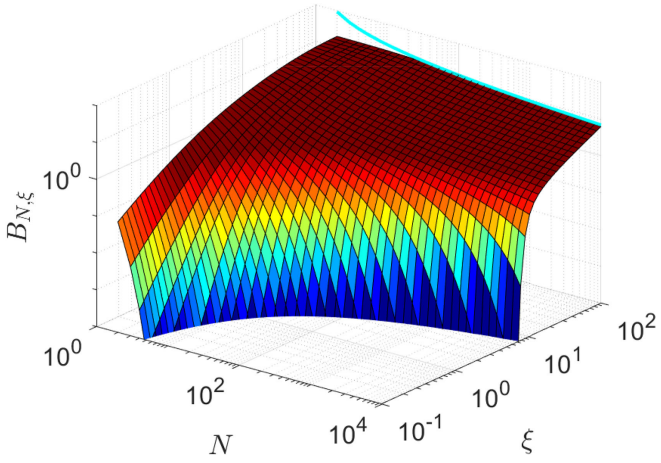

FIG. 1. 3D plot of $B_{N,\xi}$ obtained from numerical evaluation of the integrals in Eqs. (6a) and (6b), as function of N and ξ. The cyan solid line indicates our asymptotic large-ξ prediction $2\xi/(\pi \ln N)$, see Eqs. (10) and (12).

and 10^3 $\mu m^2/s$ [41], respectively, such that $\xi \sim 10^4 x_0/\mu m$ or even $\xi \sim 10^5 x_0/\mu m$. For x_0 of the order of a few micrometers, ξ is therefore very large. In turn, in a crowded environment, e.g., the cellular cytoplasm, the correlation time of noise is much longer: $(2\lambda)^{-1}$ typically ranges from milliseconds to seconds [42,43]. Concurrently, the diffusion coefficient of a small protein in cytoplasm is typically suppressed by more than an order of magnitude compared to water [44] and the ionic diffusion is reduced by a factor of a few [41]. Respectively, one finds $\xi \sim (1\text{–}10) x_0/\mu m$. Again, for x_0 in the range of a few micrometers, ξ should attain in this physical situation modest values. Last, consider the dynamics of bacteria such as *E. coli*, which performs paradigmatic run-and-tumble motion [31,32]. A systematic analysis of the experimental data has been recently performed in [33], giving $D \approx 0.3$ $\mu m^2/s$, $v \approx 17$ $\mu m/s$ and the mean run time of order of 3 s. This leads to $\xi \approx 0.5 x_0/\mu m$, and hence, for a target site placed at a distance of a few micrometers away, ξ amounts to several units. Therefore, all values of ξ are in principle relevant but correspond to different physical situations: while ξ can be large for the dynamics in aqueous solutions, moderate or even relatively small values correspond to the motion of bacteria, or ions and small proteins in cellular cytoplasm.

Asymptotic large-N form of $B_{N,\xi}$—We first consider the regime when ξ is fixed and $N \gg N_\xi$, where

$$N_\xi = -1/\ln(1 - e^{-\xi}), \tag{7}$$

a crucial parameter setting the scale for the number of searchers beyond which the exponential decay becomes relevant. We show in Supplemental Material, Sec. B2 [39] that in this regime

$$B_{N,\xi} \approx 2(e^\xi - 1)(\xi^2 N)^{-1} e^{-N/N_\xi} \quad (N \gg N_\xi), \tag{8}$$

which demonstrates that the large-N decay of $B_{N,\xi}$ is always *exponential*, in striking contrast with the prediction in Eq. (1) for Brownian walkers. This behavior agrees with the earlier prediction in [24], which was obtained in a different way. In Fig. 2, we depict the asymptotic form in Eq. (8) for different values of ξ together with the corresponding values of N_ξ, alongside numerical evaluations of the integrals in Eqs. (6a) and (6b). We observe excellent agreement between Eq. (8) and the numerical evaluation of the excess factor $B_{N,\xi}$.

Intermediate-N form of $B_{N,\xi}$—By virtue of Eq. (7), the threshold value is $N_\xi \simeq e^\xi$ for large ξ, so that the range in which the large-N exponential behavior can be observed shifts to extremely large (and even unphysical) values of N. Indeed, while N_ξ has quite modest values for small ξ (e.g., $N_\xi \approx 2.18$ for $\xi = 1$, $N_\xi \approx 6.88$ for $\xi = 2$, and $N_\xi \approx 148$ for $\xi = 5$), already for $\xi = 26$ it exceeds 10^{11}, beyond practically realizable numbers of deployed searchers. Therefore, the intermediate-N regime $3 \le N \ll N_\xi$ is a physically relevant domain for systems in which ξ is large, e.g., in aqueous solutions.

The asymptotic analysis in this regime is much subtler. In Supplemental Material, Sec. B3 [39], we show that $B_{N,\xi}$ can be accurately approximated in this range of N as

$$B_{N,\xi} \approx \int_0^1 \frac{1 - x^4}{x^3} [\mathrm{erf}(\sqrt{\xi} x)]^N dx, \tag{9}$$

where $\mathrm{erf}(x)$ is the error function. In Fig. 2 we depict the approximate form (9) by empty colored circles, confronting this prediction with the numerically evaluated integrals in Eqs. (6a) and (6b). We observe perfect agreement over an extended range of N, which broadens progressively as ξ

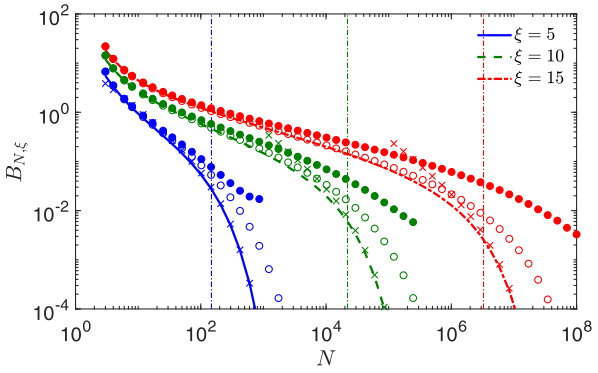

FIG. 2. Excess factor $B_{N,\xi}$ for $\xi = 5$ (blue), $\xi = 10$ (green), and $\xi = 15$ (red) as function of N. Colored solid, dashed, and dash-dotted curves depict the corresponding factors $B_{N,\xi}$ obtained by a numerical evaluation of the integrals in Eqs. (6a) and (6b). Crosses present the large-N asymptotic form in Eq. (8). Empty colored circles show the intermediate-N asymptotic form in Eq. (9), whereas the filled circles indicate the asymptotic relation (10). Colored vertical thin dash-dotted lines represent $N_\xi \approx 148$, $N_\xi \approx 2.2 \times 10^4$, and $N_\xi \approx 3.3 \times 10^6$, respectively.

increases. Furthermore, using the tight upper bound on $\mathrm{erf}(x)$ [45] and assuming that N is even, we find the following asymptotic relation which holds uniformly for any N within the interval $4 \le N \ll N_\xi$,

$$B_{N,\xi} \approx \frac{2\xi}{\pi} S_{N/2} - 1 + \frac{\pi}{8\xi} H_{N/2} + \mathcal{O}(e^{-\xi}), \tag{10}$$

where $H_n = \sum_{j=1}^{n} 1/j$ is the harmonic number, and

$$S_n = \sum_{j=1}^{n} (-1)^j \binom{n}{j} j \ln(j). \tag{11}$$

The above result is presented in Fig. 2 by filled colored circles and shows that Eq. (10) provides an accurate approximation of the behavior of the excess factor $B_{N,\xi}$ for intermediate values of N. In the limit $\xi \to \infty$, one has $N_\xi \to \infty$, and we may consider the asymptotic behavior of $B_{N,\xi}$ when $N \to \infty$. In this limit the dominant contribution to $B_{N,\xi}$ in Eq. (10) comes from the first term proportional to ξ. Taking into account that (see Supplemental Material, Sec. B3 [39])

$$S_{N/2} \simeq \frac{1}{\ln(N/2)} - \frac{\gamma}{\ln^2(N/2)} + \frac{\gamma^2 + \pi^2/6}{\ln^3(N/2)}, \tag{12}$$

where $\gamma = 0.577\ldots$ is the Euler-Mascheroni constant, and using Eq. (10), we find that, to leading order in N,

$$\overline{\mathcal{T}_N} \simeq \frac{x_0^2}{\pi D \ln N}. \tag{13}$$

This expression agrees with Eq. (1) for Brownian walkers apart from the numerical factor ($1/\pi$ instead of $1/4$). The function $2\xi/[\pi \ln(N)]$ is depicted by the thick solid line in Fig. 1 showing a very good agreement with the numerically evaluated $B_{N,\xi}$. For a discussion of how one recovers Eq. (1) as the diffusion limit of the dichotomous noise, see Supplemental Material, Sec. B4 [39].

It is legitimate to ask whether the mean fFPT is representative of the actual behavior. To provide an answer, we consider the coefficient of variation κ of the fFPT, defined as the ratio of the standard deviation of the realization-dependent fFPT divided by its mean,

$$\kappa = \frac{\sqrt{\overline{\mathcal{T}_N^2} - \overline{\mathcal{T}_N}^2}}{\overline{\mathcal{T}_N}}. \tag{14}$$

This quantity is illustrated in Fig. 3, where κ is shown as a function of N for three values of ξ. We find that κ exceeds unity only when a small number of searchers is deployed, which is intuitively expected. For larger N, however, κ drops sharply below unity, indicating that fluctuations of the fFPT become negligible and that the arrival times of the searchers concentrate around the common value $t_{\min}$: the mean fFPT provides a faithful characterization of the search dynamics. In fact, in the regime $N \gg N_\xi$, we find (see Supplemental Material, Sec. B5 [39]) that κ decays *exponentially* with N,

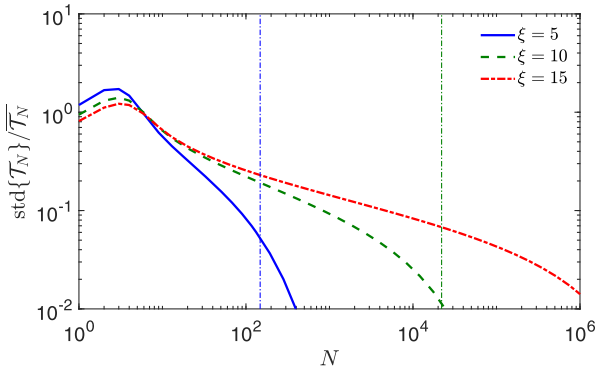

FIG. 3. Coefficient of variation κ from Eq. (14) as function of N for three values of ξ.

$$\kappa \approx \frac{2\sqrt{2}(e^\xi - 1)}{\xi^2 N} e^{-N/(2N_\xi)}. \tag{15}$$

Note that this result can also be derived from the asymptotic analysis of Ref. [24].

Anomalous diffusion—To extend the applicability of our analysis, we also consider anomalous diffusions with the MSD scaling $\overline{x^2(t)} \propto t^\alpha$ at long times, where the scaling exponent α may be less than unity (subdiffusion) or greater than unity (superdiffusion) [22,46,47]. In contrast to Brownian motion, anomalous diffusions are not universal in the sense that the same scaling exponent can originate from many physically different stochastic processes such as, e.g., fractional Brownian motion [48–51], generalized Langevin equation with memory kernels [52], or fractional diffusion equation [53]. Being driven by Gaussian noises, these stochastic processes produce the mean fFPT that is expected to inherit the anomalously slow decrease with N, like Eq. (2) for the case of the fractional diffusion equation [23]. In contrast, we consider a Riemann-Liouville-type fractional dichotomous process, introduced in [54], in which individual searcher trajectories are described by the stochastic integral

$$x_k(t) = x_0 + \int_0^t K(t-t')\eta_k(t')dt',$$
$$K(t) = (t/T_0)^{(\alpha-1)/2}/\Gamma[(\alpha+1)/2], \qquad 0 < \alpha < 2, \tag{16}$$

where T_0 is a characteristic timescale of the power-law memory kernel $K(t)$. The MSD of such a process grows with time as $\overline{x^2(t)} \propto t^\alpha$, similarly to the fractional dynamics studied in [23]. In a sharp contrast to it, the centered process

$y_k(t) = x_k(t) - x_0$ evolves inside the "light cone" $|y_k(t)| \leq y^*(t)$ for any t, where $y^*(t) \sim vT_0(t/T_0)^{(1+\alpha)/2}$. The position PDF of $y_k(t)$, which is supported on $[-y^*(t), y^*(t)]$, was studied in detail in [54].

As the above process preserves the essential property of finite-speed propagation, it is a convenient unifying framework to illustrate the consequences for the search efficiency in the N-searcher case. In Supplemental Material, Sec. C [39], we deduce the minimal travel time

$$t_{\min} = T_0 \left(\frac{x_0 \Gamma[(3+\alpha)/2]}{vT_0} \right)^{2/(1+\alpha)}. \tag{17}$$

If the timescale T_0 of the memory kernel is sufficiently small to ensure the condition $x_0/(vT_0) > 2.16$, we show that $t_{\min}$ is a *decreasing* function of $\alpha \in (0, 2)$. In other words, $t_{\min}$ is smaller in the superdiffusive regime than in the diffusive one, and that one is again less than for subdiffusion—a trend which is apparent in Fig. 4. Moreover, we observe that the approach of the mean fFPT to $t_{\min}$ as N increases is fastest for superdiffusion and longest for subdiffusion, which agrees with physical intuition. A deeper theoretical analysis of this model and its validation against experimental data for various systems may further clarify search efficiency in biology.

Conclusions—We revisited the biologically relevant problem of parallel search by N independent random searchers for an immobile target, a paradigmatic setting for extreme first-passage statistics in many applications. In the standard Brownian-motion framework, the mean fFPT decreases only logarithmically with N and formally vanishes as $N \to \infty$, implying both instantaneous arrival in the infinite-N limit and a weak benefit from deploying multiple searchers.

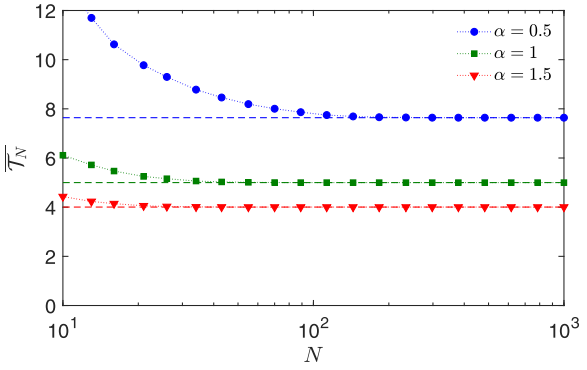

FIG. 4. Mean fFPT $\overline{\mathcal{T}_N}$ as a function of N for the dynamics generated by Riemann-Liouville dichotomous noise for three values of the scaling exponent: $\alpha = 0.5$ (subdiffusive behavior, blue), $\alpha = 1$ (diffusive behavior, green), and $\alpha = 1.5$ (superdiffusive behavior, red). The symbols depict the results of simulations with $M = 10^4$ particles, for $T_0 = 1$, $D = 1$, $v = 1$, $\lambda = v^2/(2D)$, and $x_0 = 5$ [such that $x_0/(vT_0) > 1$]. The horizontal dashed lines indicate the corresponding ballistic travels times in Eq. (17).

To regularize the unphysical short-time behavior of Brownian motion, we considered searchers driven by dichotomous noise. The resulting telegrapher's dynamics possesses a finite propagation speed, thereby eliminating arbitrarily fast arrivals. As a consequence, the mean fFPT converges in the limit $N \to \infty$ to the minimal physically attainable search time, namely, the ballistic travel time to the target. The convergence to this value with increasing N is entirely controlled by the key parameter—the dimensionless ballistic travel time $\xi = x_0\lambda/v$. The parameter ξ can be quite small for particles moving in viscoelastic media such as the cell cytoplasm, or for bacteria, but it may attain large values for particles moving in aqueous solutions. We show that ξ determines the characteristic number N_ξ that separates two asymptotic regimes: (i) the exponential decay for $N \gg N_\xi$, and (ii) a much slower decay for $3 \leq N \ll N_\xi$, which, for $\xi \to \infty$ (and hence $N_\xi \to \infty$) becomes consistent with the dependence in Eq. (1), up to a numerical prefactor. Concurrently, the result in Eq. (1) follows directly from our analysis if we first take the "diffusion limit" of the dichotomous noise and then consider the limit $N \to \infty$, which demonstrates that these limits do not commute (see also Ref. [55]).

In the first regime, the exponential convergence to the lower bound demonstrates a dramatic efficiency gain over Brownian search in the way how rapidly each added searcher reduces the mean fFPT. From an evolutionary perspective, if the random searchers exhibit finite-speed motion with moderate ξ, it is advantageous to invest the energy to produce many searchers to substantially speed up underlying random search processes. For intermediate N, the convergence remains slow and mimics the familiar logarithmic behavior, thereby reconciling earlier results with finite-speed dynamics. It is worth noting that biological systems operate under various constraints and eventual costs so that the actual origin of redundancy can be outside the minimization of the mean fFPT.

We extended our analysis to anomalous transport via the Riemann-Liouville dichotomous process. This yielded a clear hierarchy: superdiffusive search is most efficient, followed by diffusive scaling and then subdiffusion, in contrast to predictions from the fractional diffusion equation.

Overall, our results show that imposing a finite propagation speed fundamentally alters the extreme first-passage behavior, resolves long-standing paradoxes of Brownian statistics, and identifies the dramatic gain of using multiple finite-speed seachers for rapid target detection. Moreover, as the dependence of the mean fFPT on N is very sensitive to the short-time dynamics of a single searcher, which is nonuniversal and system-dependent, a formal extension of the extreme-value statistics from Brownian particles to biological searchers can be strongly misleading. A systematic comparison of our predictions with other

microscopic models exhibiting minimal travel times (e.g., discrete-time random walks on lattices or graphs [55]) and with experimental data will provide further insight into redundancy and optimality in collective search.

Acknowledgments—The authors thank Dr. S. Lawley for bringing our attention to his paper [24]. D. S. G. acknowledges partial support by the Alexander von Humboldt Foundation within a Bessel Prize award. R. M. acknowledges Grants No. ME 1535/22-1 and No. 318763901, CRC1294 Project No. B10, from the German Science Foundation (DFG).

Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

[1] M. Ptashne and A. Gann, *Genes and Signals* (Cold Spring Harbor Laboratory Press, New York, 2002).
[2] B. Alberts, A. Johnson, J. Lewis, M. Raff, K. Roberts, and P. Walter, *Molecular Biology of the Cell* (Garland Science, New York, 2002).
[3] O. Bénichou, C. Loverdo, M. Moreau, and R. Voituriez, Intermittent search strategies, Rev. Mod. Phys. **83**, 81 (2011).
[4] P. Hammar, P. Leroy, A. Mahmutovic, E. G. Marklund, O. G. Berg, and J. Elf, The lac repressor displays facilitated diffusion in living cells, Science **336**, 1595 (2012).
[5] J. Ma, M. Do, M. A. Le Gros, C. S. Peskin, C. A. Larabell, Y. Mori, and S. A. Isaacson, Strong intracellular signal inactivation produces sharper and more robust signaling from cell membrane to nucleus, PLoS Comput. Biol. **16**, e1008356 (2020).
[6] M. Bauer and R. Metzler, In vivo facilitated diffusion model, PLoS One **8**, e53956 (2013).
[7] D. P. Snustad and M. J. Simmons, *Principles of Genetics* (Wiley, New York, 2015).
[8] *Target Search Problems*, edited by D. S. Grebenkov, R. Metzler, and G. Oshanin (Springer, Cham, CH, 2024).
[9] G. H. Weiss, K. E. Shuler, and K. Lindenberg, Order statistics for first passage times in diffusion processes, J. Stat. Phys. **31**, 255 (1983).
[10] B. Meerson and S. Redner, Mortality, redundancy, and diversity in stochastic search, Phys. Rev. Lett. **114**, 198101 (2015).
[11] K. Reynaud, Z. Schuss, N. Rouach, and D. Holcman, Why so many sperm cells?, Comm. Int. Biol. **8**, e1017156 (2015).
[12] S. D. Lawley, Universal formula for extreme first passage statistics of diffusion, Phys. Rev. E **101**, 012413 (2020).
[13] S. D. Lawley, Distribution of extreme first passage times of diffusion, J. Math. Biol. **80**, 2301 (2020).
[14] S. D. Lawley and J. B. Madrid, A probabilistic approach to extreme statistics of Brownian escape times in dimensions 1, 2, and 3, J. Nonlinear Sci. **30**, 1207 (2020).
[15] J. B. Madrid and S. D. Lawley, Competition between slow and fast regimes for extreme first passage times of diffusion, J. Phys. A **53**, 335002 (2020).
[16] Z. Schuss, K. Basnayake, and D. Holcman, Redundancy principle and the role of extreme statistics in molecular and cellular biology, Phys. Life Rev. **28**, 52 (2019).
[17] T. Baravi and E. Barkai, The thermodynamic limit of extreme first-passage times, J. Stat. Mech. (2025) P123205.
[18] D. S. Grebenkov, R. Metzler, and G. Oshanin, From single-particle stochastic kinetics to macroscopic reaction rates: Fastest first-passage time of N random walkers, New J. Phys. **22**, 103004 (2020).
[19] D. S. Grebenkov, R. Metzler, and G. Oshanin, Search efficiency in the Adam-Delbrück reduction-of-dimensionality scenario versus direct diffusive search, New J. Phys. **24**, 083035 (2022).
[20] A. Godec and R. Metzler, Universal proximity effect in target search kinetics in the few encounter limit, Phys. Rev. X **6**, 041037 (2016).
[21] D. S. Grebenkov, R. Metzler, and G. Oshanin, Strong defocusing of molecular reaction times: Geometry and reaction control, Commun. Chem. **1**, 96 (2018).
[22] R. Metzler, J.-H. Jeon, A. G. Cherstvy, and E. Barkai, Anomalous diffusion models and their properties: Non-stationarity, non-ergodicity, and ageing at the centenary of single particle tracking, Phys. Chem. Chem. Phys. **16**, 24128 (2014).
[23] S. D. Lawley, Extreme statistics of anomalous subdiffusion following a fractional Fokker-Planck equation: Subdiffusion is faster than normal diffusion, J. Phys. A **53**, 385005 (2020).
[24] S. D. Lawley, Extreme first passage times of piecewise deterministic Markov processes, Nonlinearity **34**, 2750 (2021).
[25] C. R. Cattaneo, Sur une forme de l'équation de la chaleur éliminant le paradoxe d'une propagation instantanée (On a form of the heat equation eliminating the paradox of an instantaneous propagation), C.R. Hebd. Seances Acad. Sci. **247**, 431 (1958).
[26] D. Jou, J. Casas-Vázquez, and G. Lebon, *Extended Irreversible Thermodynamics* (Springer, New York, 2001).
[27] P. Hänggi and P. Jung, Colored noise in dynamical systems, Adv. Chem. Phys. **89**, 239 (1995).
[28] I. Bena, Dichotomous Markov noise: Exact results for out-of-equilibrium systems, Int. J. Mod. Phys. B **20**, 2825 (2006).
[29] T. Sandev, L. Kocarev, R. Metzler, and A. Chechkin, Stochastic dynamics with multiplicative dichotomic noise: Heterogeneous telegrapher's equation, anomalous crossovers and resetting, Chaos Solitons Fractals **165**, 112878 (2022).
[30] T. Herbeau, L. Pastur, P. Viot, and G. Oshanin, Stochastic gyration driven by dichotomous noises, J. Stat. Mech. (2026) P013205.
[31] J. Tailleur and M. E. Cates, Statistical mechanics of interacting run-and-tumble bacteria, Phys. Rev. Lett. **100**, 218103 (2008).
[32] A. B. Slowman, M. R. Evans, and R. A. Blythe, Jamming and attraction of interacting run-and-tumble random walkers, Phys. Rev. Lett. **116**, 218101 (2016).
[33] Y. Zhao, C. Kurzthaler, N. Zhou, J. Schwarz-Linek, C. Devailly, J. Arlt, J.-D. Huang, Wilson C. K. Poon, T. Franosch, V. A. Martinez, and J. Tailleur, Quantitative

characterization of run-and-tumble statistics in bulk bacterial suspensions, Phys. Rev. E **109**, 014612 (2024).
[34] O. Bénichou, P. Illien, G. Oshanin, A. Sarracino, and R. Voituriez, Tracer diffusion in crowded narrow channels, J. Phys. Condens. Matter **30**, 443001 (2018).
[35] D. J. Bicout, Greens functions and first passage time distributions for dynamic instability of microtubules, Phys. Rev. E **56**, 6656 (1997).
[36] K. Malakar, V. Jemseena, A. Kundu, K. Kumar, S. Sabhapandit, S. N. Majumdar, S. Redner, and A. Dhar, Steady state, relaxation and first-passage properties of a run-and-tumble particle in one-dimension, J. Stat. Mech. (2018) P043215.
[37] F. Mori, P. Le Doussal, S. N. Majumdar, and G. Schehr, Universal survival probability for a d-dimensional run-and-tumble particle, Phys. Rev. Lett. **124**, 090603 (2020).
[38] U. Basu, S. Sabhapandit, and I. Santra, *Target Search by Active Particles, in [8]* (Springer, Cham, 2024), p. 463.
[39] See Supplemental Material at http://link.aps.org/supplemental/10.1103/7q39-h9dv for additional details on the exact solution of the boundary value problem for a single particle, the calculation of the fastest first-passage time and its detailed properties, and the simulation of anomalous-diffusion dynamics.
[40] M. Yu, T. Castanheira Silva, A. van Opstal, S. Romeijn, H. A. Every, W. Jiskoot, G.-J. Witkamp, and M. Ottens, The investigation of protein diffusion via H-cell microfluidics, Biophys. J. **116**, 595 (2019).
[41] P. Swietach, M. Zaniboni, A. K. Stewart, A. Rossini, K. W. Spitzer, and R. D. Vaughan-Jones, Modelling intracellular H(+) ion diffusion, Prog. Biophys. Molec. Biol. **83**, 69 (2003).
[42] T. J. Lampo, S. Stylianidou, M. P. Backlund, P. A. Wiggins, and A. J. Spakowitz, Cytoplasmic RNA-protein particles exhibit non-Gaussian subdiffusive behavior, Biophys. J. **112**, 532 (2017).
[43] C. Di Rienzo, V. Piazza, E. Gratton, F. Beltram, and F. Cardarelli, Probing short-range protein Brownian motion in the cytoplasm of living cells, Nat. Commun. **5**, 5891 (2014).
[44] O. Seksek, J. Biwersi, and A. S. Verkman, Translational diffusion of macromolecule-sized solutes in cytoplasm and nucleus, J. Cell Biol. **138**, 131 (1997).
[45] J. D. Williams, An approximation to the probability integral, Ann. Math. Stat. **17**, 363 (1946).
[46] G. Muñoz-Gil, G. Volpe, M. A. Garcia-March, E. Aghion, A. Argun, C. B. Hong *et al.*, Objective comparison of methods to decode anomalous diffusion, Nat. Commun. **12**, 6253 (2021).
[47] H. Seckler and R. Metzler, Bayesian deep learning for error estimation in the analysis of anomalous diffusion, Nat. Commun. **13**, 6717 (2022).
[48] B. B. Mandelbrot and J. W. Van Ness, Fractional Brownian motions, fractional noises and applications, SIAM Rev. **10**, 422 (1968).
[49] Y. S. Mishura, *Stochastic Calculus for Fractional Brownian Motion and Related Processes* (Springer-Verlag, Berlin, 2008).
[50] O. Vilk, E. Aghion, T. Avgar, C. Beta, O. Nagel, A. Sabri, R. Sarfati, D. K. Schwartz, M. Weiss, D. Krapf, R. Nathan, R. Metzler, and M. Assaf, Unravelling the origins of anomalous diffusion: from molecules to migrating storks, Phys. Rev. Res. **4**, 033055 (2022).
[51] W. Wang, Q. Wei, A. V. Chechkin, and R. Metzler, Different behaviors of diffusing diffusivity dynamics based on three different definitions of fractional Brownian motion, Phys. Rev. E **112**, 014108 (2025).
[52] S. C. Kou, Stochastic modeling in nanoscale biophysics: Subdiffusion within proteins, Ann. Appl. Stat. **2**, 501 (2008).
[53] R. Gorenflo and F. Mainardi, Subordination pathways to fractional diffusion, Eur. Phys. J. Special Topics **193**, 119 (2011).
[54] D. S. Dean, S. N. Majumdar, and H. Schawe, Position distribution in a generalized run-and-tumble process, Phys. Rev. E **103**, 012130 (2021).
[55] S. D. Lawley, Extreme first-passage times for random walks on networks, Phys. Rev. E **102**, 062118 (2020).