Fractional Calculus: An Introduction for Physicists

Ralf Metzler

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parallelism notion mentioned earlier; and section 13.9, on the role of entanglement in the power of quantum computing, a topic still poorly understood and certainly less obvious than one might expect.

And now, I've just about reached my word limit. But I have space to repeat the key one: masterpiece. I need not say more.

Valerio Scarani
National University of Singapore

Fractional Calculus

An Introduction for Physicists

Richard Herrmann
World Scientific, Hackensack, NJ, 2011. $85.00 (261 pp.).
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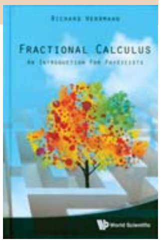

Essentially any account of fractional calculus starts with the mention of Gottfried Wilhelm Leibniz. In a letter to French mathematician Guillaume de l'Hôpital dated 1695, Leibniz asked what meaning one might assign to a non-integer-order differential. The scientists who followed up on Leibniz's question read like a *Who's Who* of mathematics: They include Jacques Hadamard, Paul Pierre Lévy, Joseph Liouville, Bernhard Riemann, and Hermann Weyl.

Fractional differential expressions have been used in engineering since the 1930s to describe viscoelastic materials, but their use in physics is much more recent. The idea is striking: Similar to Benoît Mandelbrot's "clouds are not spheres" appeal, which popularized fractal geometry involving non-integer spatial dimensions, fractional derivatives and integrals can be applied to real systems characterized by power laws, critical phenomena, and scale-free processes.

There exists a rich selection of mathematical texts on fractional calculus, starting with the 1974 classic by Keith Oldham and Jerome Spanier, *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order* (Dover Publications, 2006). The literature on fractional calculus applications in the physical sciences is following suit, with a number of recent additions. In particular, World Scientific has devoted several books to the topic, written or edited by such experts as Rudolf Hilfer, Joseph Klafter, Francesco Mainardi, Vladimir Uckaikin, and me. The focus in most of those works is on statistical processes. In that context the occurrence of fractional derivatives with respect to space or time is directly related to continuous-time, random-walk processes with long-tailed jump-length or waiting-time distributions or, equivalently, to coupled Langevin equations mirroring the probabilistic concept of subordination to a counting process.

In *Fractional Calculus: An Introduction for Physicists*, Richard Herrmann advocates for the potential application of fractional calculus to a number of areas in the physical sciences. The book is a solid introduction to fractional calculus that contains, in particular, an elucidating section on the geometric interpretation of fractional operators. Contrary to its own blurb, the bulk of the book concentrates on aspects of fractional calculus related to symmetries in quantum mechanics. Curiously, the author neglects statistical mechanics and stochastic processes, the fields in which fractional methods have seen significant applications. However, what is covered is presented in an authoritative, solid style and actually provides very entertaining reading.

The author takes the reader on a journey to explore several quantum mechanical contexts to follow up on the question, What changes to the standard symmetries are effected by the introduction of fractional operators? Thus one learns about the role of parity in fractional wave equations with respect to space, how a fractional Schrödinger equation can be formulated, and the general implications of fractional spin. The author sets the scene for nuclear- and particle-physics applications, including spectra, and for nuclear magic numbers, with an in-depth introduction of the fractional rotation group. He then discusses fractional fields and their gauge invariance in the context of a fractional calculus for tensor quantities.

Fractional Calculus discusses many fascinating consequences of fractional formulations and opens up new vistas for the now conventional symmetries used in quantum and particle physics. As the author mentions, "This book is explicitly devoted to the practical consequences of using fractional calculus." However, the downside is that the book does not provide the motivation for any particular formulation. The discussed equations, for instance, are not derived as diffusion limits of random walks or from subordination arguments based on well-studied theories. More like a mathematician, Herrmann analyzes the extended equations for their properties. That ad hoc introduction of fractional operators may be appealing to casual readers, but the book's axiomatic format makes it less suitable as a text for a graduate course.

Overall, *Fractional Calculus* is an affordable and valuable introduction to the field that will appeal to physicists interested in scientific what-ifs.

Ralf Metzler
University of Potsdam
Potsdam-Golm, Germany

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