For CTRW subdiffusion in a semi-infinite domain, the $t^{-3/2}$ scaling thus changes to $p_x(t) \simeq t^{-1-\alpha/2}$

In a finite domain the exponential decay of normal diffusion turns to the Mittag-Leffler function, and $p_a(x) \propto t^{-1-\alpha}$.

VIII. LANGEVIN FORMULATION OF CTRW & EXTENSIONS. [Fogedby, PRE (1994)]

CTRW: waiting time distribution $\psi(\tau)$: $\int_0^\infty \psi(\tau)\,d\tau = 1$

jump length distribution $\lambda(x)$: $\int_{-\infty}^{\infty} \lambda(x)\,dx = 1$

Position of the walker after s steps:

$$r(s) = \sum_{s'=0}^{s} x(s') \longrightarrow \int_0^s x(s')\,ds' \text{ in the continuum limit}$$

$$\Rightarrow \frac{dr}{ds} = x(s) \quad \text{Langevin eq. with random displacement } x$$

Time elapsed after s steps:

$$t(s) = \sum_{s'=0}^{s} \tau(s') \longrightarrow t(s) = \int_0^s \tau(s')\,ds'$$

$$\Rightarrow \frac{dt}{ds} = \tau(s)$$

With external forces: $\frac{dr}{ds} = F(r) + x(s)$

Sharp waiting times: $\psi(t) = \delta(\tau - \tau_0)$

$$\frac{dt}{ds} = \tau_0 \rightsquigarrow t(s) = \tau_0 s$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{\tau_0} F(r) + \frac{1}{\tau_0} x(t)$$

Otherwise we need to consider the coupled equations for r and t