

For TA 150 we find:

$$\langle \delta^2(\Delta, t_a, T) \rangle = \frac{1}{T-\Delta} \int_{t_a+T-\Delta}^{t_a} (x(t+\Delta) - x(t))^2 dt$$

$$= \frac{\Delta_\alpha(t_a/T) g(\Delta)}{T(1-\alpha)}$$

$g(\Delta) = \Delta$ for free motion, $g(\Delta) \approx \Delta^{1-\alpha}$ under confinement

universal prefactor $\Delta_\alpha(z) = (1+z)^\alpha - z^\alpha$ for entire range of Δ !

"Death of linear response"

In presence of a time-dependent external force $F(t)$, the fractional Fokker-Planck equation becomes (Solomon & Klafki, PRL 97, 140602 (2006)):

$$\frac{\partial}{\partial t} p(x,t) = - \left(\frac{F(t)}{m\eta_\alpha} \frac{\partial}{\partial x} + K_\alpha \frac{\partial^2}{\partial x^2} \right) \mathcal{D}_t^{1-\alpha} p(x,t)$$

Considers an harmonic forcing of the form $F(t) = F_0 \sin \omega t$.

The first moment becomes

$$\langle x(t) \rangle = \frac{F_0}{m\eta_\alpha} \sqrt{2\pi} S\left(\frac{\pi}{2t}\right) \text{ in terms of the Fresnel integral } S(z) = \int_0^z \sin\left(\frac{\pi}{2} t^2\right) dt$$

Thus the particle experiences an initial shift. Due to longer & longer waiting times, it shows oscillations of decreasing amplitude.



