Trick to calculate: start with diffusion equation

$$x^2 \frac{\partial P}{\partial t} = \frac{\partial}{\partial t} x^2 P = D x^2 \frac{\partial^2 P}{\partial x^2} \quad \Big| \int_{-\infty}^{\infty} dx \,.$$

$$\frac{d}{dt}\langle x^2\rangle = D\left\{ \left[x^2 P'\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} 2x \frac{\partial P}{\partial x} dx \right\} = D\left\{ -\left[2xP\right]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} 2P\, dx \right\} = 2D$$

$\Rightarrow \langle x^2\rangle = 2Dt.$

Fick's derivation of the diffusion equation:

(1) Continuity equation: Given a probability density $P(\underline{r},t)$ with $\iiint_{-\infty}^{\infty} P(\underline{r},t)\,dV = 1$, and the probability flux $\underline{S}(\underline{r},t)$:

$$\oiint_{\delta V} \underline{S}(\underline{r},t)\, d\underline{A} = -\frac{d}{dt} \iiint_V P(\underline{r},t)\, dV = -\frac{d}{dt} \mathcal{S}(t)$$

where $\mathcal{S}(t)$ is the survival probability.
With the divergence theorem (Gauss' theorem):

$$\oiint_{\delta V} \underline{S}(\underline{r},t)\, d\underline{A} = \iiint_V \nabla\cdot \underline{S}(\underline{r},t)\, dV$$

we obtain the integral form of the continuity equation:

$$\iiint_V \nabla\cdot\underline{S}(\underline{r},t)\, dV = -\frac{d}{dt}\iiint_V P(\underline{r},t)\, dV = -\iiint_V \frac{\partial P(\underline{r},t)}{\partial t}\, dV$$

This relation is valid $\forall\ (V,\delta V) \Rightarrow$ differential form:

$$\boxed{\frac{\partial}{\partial t} P(\underline{r},t) = -\nabla\cdot\underline{S}(\underline{r},t) = -\operatorname{div}\underline{S}(\underline{r},t).}$$ continuity equation

(2) Fick's first law:

$\boxed{\underline{S} = -D\nabla P(\underline{r},t)}$ the flux is proportional to the gradient of the probability density function P.