University of Potsdam — Institute of Physics and Astronomy
Lecture Stochastic Processes (SS 2018)
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Problem Set 5

(discussion on June 6th)

α-stable distributions

Aufgabe 1 : Characteristic Functions

Determine the characteristic functions $p(k) = \langle e^{ik} \rangle$, the means and the variances of the discrete probability distributions

$$\text{(a)} \quad p_q(n) = (1-q)q^n \qquad \text{(b)} \quad p_d(n) = \frac{1}{2}\left(\delta_{-1n} + \delta_{1n}\right)$$

Aufgabe 2 : Central Limit Theorem

Using

$$\lim_{N\to\infty}\left(1 + \frac{x}{N}\right)^N = e^x$$

show that if $p(x)$ has a first moment μ and a second moment σ^2, the mean of N i.i.d. random variables $X_n - \mu$ has a distribution that tends to the Gaussian distribution of variance σ^2.

Aufgabe 3 : Holtzmark Distribution (Feller vol. 2, problem 13.7., pg.215)

We consider a uniform distribution of point masses m within a 3d sphere of radius R. Show that the z-component of the gravitational force in the center of the sphere due to N independently placed point masses is asymptotically, symmetric α-stable with a characteristic exponent of 3/2. The limit $R \to \infty$ and $N \to \infty$ is to be taken such that $\frac{4}{3}\pi R^3 N^{-1} = \lambda$. Alternatively the exponent of the limit distribution can be derived via a scaling argument by adding the forces of two point clouds with densities λ_1^{-1} and λ_2^{-1}.