University of Potsdam Institute of Physics and Astronomy
Lecture Stochastic Processes (SS 2018)
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Problem Set 1

(discussion on April 26th)

Problem 1 : conditional probabilities

Let $\{A_n \in \Sigma\}$ be a countable, complete set of mutually exclusive events, i.e. $A_n \cap A_{m\neq n} = \emptyset$ and $\bigcup_n A_n = \Omega$. Show the following completeness relations for all events $B, C \in \Sigma$:

$$\sum_n P\left(A_n|B\right) = 1 \tag{1}$$

$$\sum_n P\left(B|A_n\right) P(A_n) = P(B) \tag{2}$$

$$\sum_n P\left(B|A_n, C\right) P\left(A_n|C\right) = P\left(B|C\right) \tag{3}$$

Problem 2 : Laplace transform

(a) Determine the Laplace transform $f(u)$ for the following functions $f(t)$

$$f(t) = \delta(t - T_0), \quad f(t) = \Theta(t - T_0), \quad f(t) = e^{-\gamma t}\sin(\omega t), \quad f(t) = e^{-\gamma t}\cos(\omega t)$$

(b) Calculate the solution $x(t)$ to the initial value problem with $x(0) = x_0$, $\dot{x}(0) = v_0$ and

$$\ddot{x} + 2\gamma^2\dot{x} + \omega_0^2 x = 0$$

by first solving the linear ordinary differential equation in the Laplace domain for $x(u)$ and then transforming it back by direct coefficient comparison to the solutions of (a).

(c)* Try to solve the inhomogenous ODE with $A\cos(\Omega t)$ on the right hand side.

Problem 3 : Diffusion equation

Consider the diffusion equation

$$\partial_t \phi = D \nabla^2 \phi \tag{4}$$

for a field $\phi = \phi(x, t)$ on a domain $x \in [0, a]$ with an initial condition $\phi(x, 0) = \delta(x - x_0)$. At the domain border the field is subject to either reflecting (no flux) boundary conditions

$$\partial_x \phi(a, t) = \partial_x \phi(0, t) = 0 \tag{5}$$

or absorbing boundary conditions

$$\phi(a, t) = \phi(0, t) = 0 \tag{6}$$

Find the solutions of (4) under each of these boundary conditions unsing a separation ansatz and orthonormal eigenfunctions approach.