

# Extreme Value Statistics

## Assignments #2

July 9, 2007

1. A return level for probability  $p$  is defined as the quantile  $z_p$  such that  $G(z_p) = p$ . Give the expression for  $z_p$  as a function of location, scale and shape parameter  $(\mu, \sigma, \xi)$  for the GEV:

$$G(z) = \exp \left\{ - \left[ 1 + \xi \left( \frac{z - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}. \quad (1)$$

Hint: consider two cases: a)  $\xi = 0$  and b)  $\xi \neq 0$ , and use the expression for the Gumbel distribution for a):

$$G_G(z) = \exp \left\{ - \exp \left[ - \left( \frac{z - b}{a} \right) \right] \right\}. \quad (2)$$

2. Perform an extreme value analysis for the Elbe annual maxima using the GEV.
  - (a) Use R to analyse the data, either on the local machines (it works on those two with SuSE 9.3) or log-in to voltaire as last time with the `guestn` accounts.
  - (b) if you are local, copy the following file in your directory:  
`cp ~hrust/Lab/.emacs ./`, on voltaire it is present for all guest accounts.
  - (c) copy the data file: `cp ~hrust/Lab/Assignment2/ElbeMaxima.dat ./`
  - (d) open a new R-script file with: `emacs Elbe-Analysis.R &`
  - (e) emacs opens hopefully in ESS mode (see status line at the bottom), use `CTRL-x 2` to open a second window, click inside and type `ALT-x R` and press return, R starts up in the second window.
  - (f) Now, write the code for your analysis in the first window and copy and paste the lines into R (works also with mouse marking and the buttons from the tool bar).
  - (g) first, load some libraries and additional functions:  

```
> library(evd)
```
  - (h) read in the data file:  

```
> maxima <- read.table("ElbeMaxima.dat",header=TRUE)
```
  - (i) estimate GEV parameters  

```
> gev.fit <- fgev(maxima$Qmax)
```
  - (j) look at the result with  

```
> gev.fit
```

What are the location, scale and shape parameters? What are their approximated standard errors? Use the standard errors and asymptotic normality of the estimator to construct a 95% confidence interval for each of the three parameters. Hint: the corresponding quantiles of the normal distribution can be found at <http://www.math.uni-magdeburg.de/~brueckne/kb/files/ws03/figure47.pdf>.

- (k) plot PP, QQ, density and return-level plots to a PDF.

```
> pdf("ElbeGEVFit.pdf")
> layout(matrix(1:4,ncol=2))
> plot(gev.fit)
> dev.off()
```

What do they represent? What is the difference between a probability plot (PP) and a quantile plot (QQ)?

- (l) the shape parameter  $\xi$  is compatible with 0, i.e. 0 is included in the 95% CI. This indicates that the Gumbel distribution might be more adequate than the full GEV. Repeat the fit forcing the shape parameter  $\xi = 0$ , i.e. fit a Gumbel distribution:

```
> gumbel.fit <- fgev(maxima$Qmax,shape=0)
```

- (m) compare location and scale parameter and their standard errors to the estimates for the full GEV. What is the difference? Generate also the plots.
- (n) use the result from l) to obtain a 100-year return level from the parameters estimated for the Gumbel distribution. Hint: calculation in R is straight forward: `result <- a + b * log(1-c)` etc.
- (o) standard errors  $\sigma_{z_p}$  for return levels  $z_p$  can be obtained using the delta method. In case of the Gumbel distribution this is:

$$\sigma_{z_p}^2 = \left( \frac{\delta}{\delta\mu} z_p(\mu, \sigma) \right)^2 \sigma_\mu^2 + \left( \frac{\delta}{\delta\sigma} z_p(\mu, \sigma) \right)^2 \sigma_\sigma^2.$$

Use this result to obtain a 95% CI for the 100-year return level. Hint: the fit results can be accessed as `gumbel.fit$estimate[1]` for the location and `gumbel.fit$estimate[2]` for the scale parameter. Standard errors are in `gumbel.fit$std.err`.