Extreme Value Statistics

Assignments #1

June 28, 2007

1. Show that the Fréchet distribution (Type II)

$$G_F(z) = \begin{cases} 0 & , \quad z \leq b \\ \exp\left\{-\left(\frac{z-b}{a}\right)^{-\alpha}\right\} & , \quad z > b \end{cases} \tag{1}$$

and Weibull distributions (Type III)

$$G_W(z) = \begin{cases} \exp\left\{-\left[-\left(\frac{z-b}{a}\right)^{\alpha}\right]\right\} & , \quad z > b \\ 1 & , \quad z \geq b \end{cases} \tag{2}$$

are special cases of the general extreme value distribution (GEV).

$$G(z) = \exp\left\{-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-1/\xi}\right\} \tag{3}$$

defined on $\{z \mid 1+\xi(z-\mu)/\sigma > 0\}$.
Extra task: show that the Gumbel distribution

$$G_G(z) = \exp\left\{-\exp\left[-\left(\frac{z-b}{a}\right)\right]\right\} \tag{4}$$

with $-\infty < z < \infty$ results as the limit of the GEV for $\xi \to 0$.

2. Show that the Gumbel distribution is max-stable, i.e.

$$G_G^n(\alpha_n z + \beta_n) = G_G(z) \tag{5}$$

for every $n = 2, 3, \ldots$ and $\alpha_n > 0$.

3. Get familiar with `R`: install it (or use the lab computers), find a suitable text editor to write scripts (I recommend emacs and the ESS-package[1], use the accompanying `R`-script `evs_assignments1.R` and the data file `evs_assignments1.dat` as a start. Copy and paste from a text editor into the console of `R`, play with the parameters of

   - `hist()`, use `help(hist)` to get more information on it and
   - `density()`, try different kernels and bandwidth
   - generate samples of different random variables X and look at the distribution of the maxima M_n for different n

[1] `http://ess.r-project.org/`, unfortunately not in the lab.