University of Potsdam, Institute for Physics	SS 2005
Linear and Nonlinear Methods for Time Series Analysis	
Lectures on Friday 11:00-12:30, room 1.19.415	J. Kurths
Exercise in room 1.19.423 (Computerpool)	U. Schwarz
Group 1 on Tuesday 9:30-11:00	Group 2 on Thursday 12:45-14:15

Exercise 1, Due date 3.05.2005

Question 1: Consider the binomial distribution with $n = 6$ und $P = 1/4$. Tabulate the probabilities for the different possible values. Plot the distribution function F and compute the moments μ and σ^2 directly from the probabilities. Do they agree with the theoretical formula $\mu = n\ P$ and $\sigma^2 = n\ P\ (1 - P)$?
(2 P.)

Question 2: The two dimensional normal distribution in (X, Y) has the density function

$$f(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2 - 2\rho\frac{x-\mu_X}{\sigma_X}\frac{y-\mu_Y}{\sigma_Y} + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2\right]\right\}$$

with the parameters μ_X, μ_Y, σ_X, σ_Y and ρ. Find the marginal density for X und Y – Which distribution so they follow ? –, and from the result show that X and Y are independent when $\rho = 0$.
(3 P.)

Question 3: **a**] The empirical variance s^2 a sample $\vec{x}$ with length n, considered as an estimator function $\psi(\vec{X})$ of the random vector $\vec{X}$, is given by

$$\hat{S}_n^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2 \quad \text{mit} \quad \bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

Plot $\bar{X}_n$ and $\hat{S}_n^2$ for a chosen sample length with uniform and Gaussian noise. Discuss this result. Attach the source code.
(4 P.)

b] Show that for any random variable X with variance σ^2 we have

$$\langle \hat{S}_n^2 \rangle = \frac{n-1}{n}\ \sigma^2$$

This means that the estimator $\langle \hat{S}_n^2 \rangle$ of σ^2 is biased. *Hint: To make the calculation as clear as possible please keep in mind that* $\langle\ \rangle$ *is a linear operator.*
(2 P.)