Determination of fixed points inside wind fields

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- Method
- Testing

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Motivation

- General Circulation Models:
  - Data on a rough lattice
  - resolution about 1° to 5°
- Measuring data at some discrete points only
- Often additional noise

⇒ Analysing a methodology to find fixed points inside wind fields given as discrete data sets concerning resolution and level of noise

The data

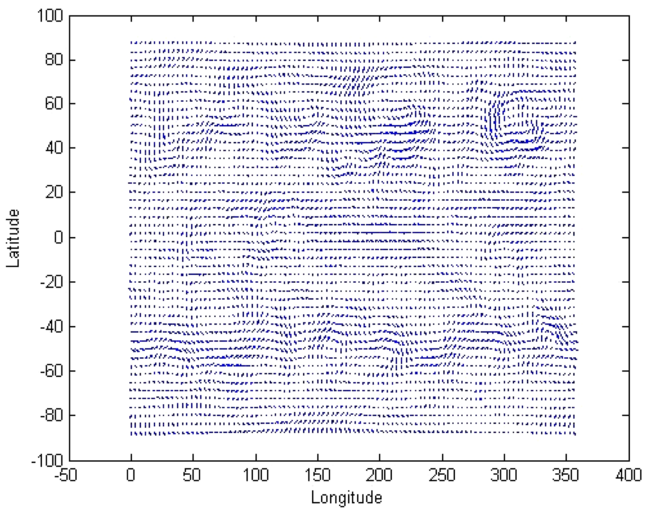
- Aim: analysis of wind fields (fixed point)
- Data source: ECHO-GiSP GCM

  ECHAM & HOPE-G including Stratosphere by AWI Research Unit Potsdam

  Netcdf data
  - resolution: Longitude 3.75° (417 km)
    Latitude approx. 3.71° (412 km)
  - 23 layer: not linear spaced, 1 Pa to 100000 Pa (40 km to 0.1 km)
  - 95 years, semi-daily

- Use Matlab

Wind field



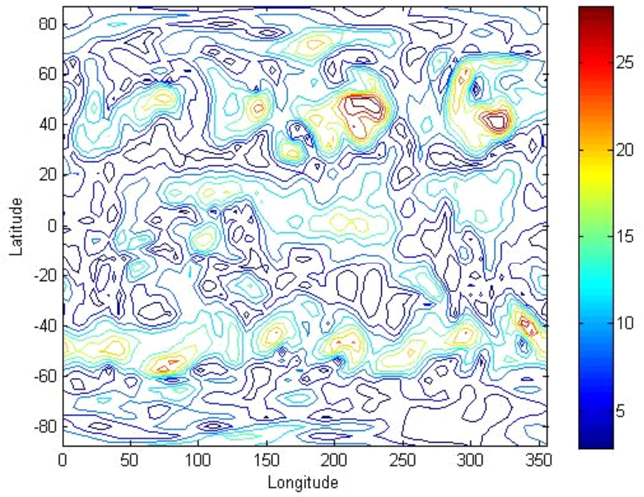
- Horizontal components

  $\rightarrow \mathbb{R}^2$ instead $\mathbb{R}^3$
- Treat each layer separately
- Neglect vertical wind (generally weaker)

Wind speed

Example:
January 1985, lower troposphere

Absolute value of
horizontal wind in m/s



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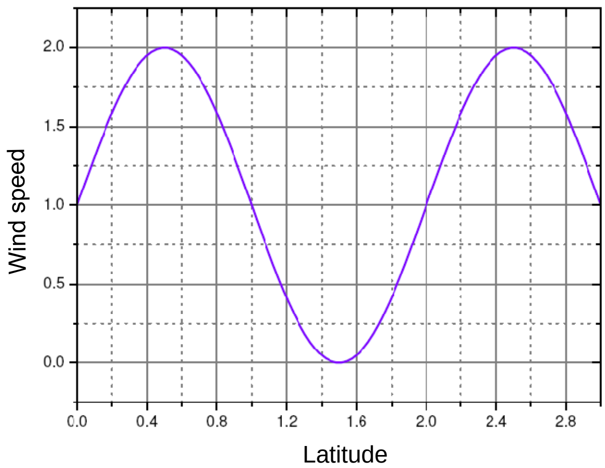
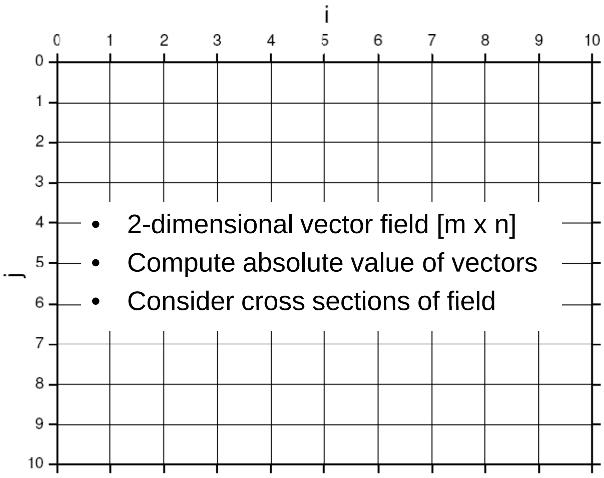
Idea

- Condition:
  - Velocity at position of fixed point is zero
  - Through discretization minima instead of value zero



Method



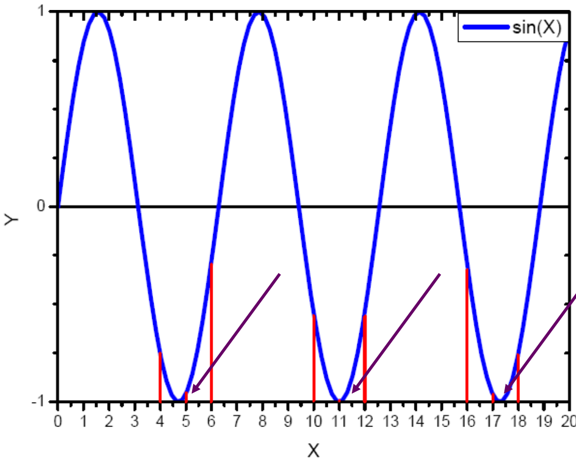
- 2-dimensional vector field [m x n]
- Compute absolute value of vectors
- Consider cross sections of field

⇒ 1d problem

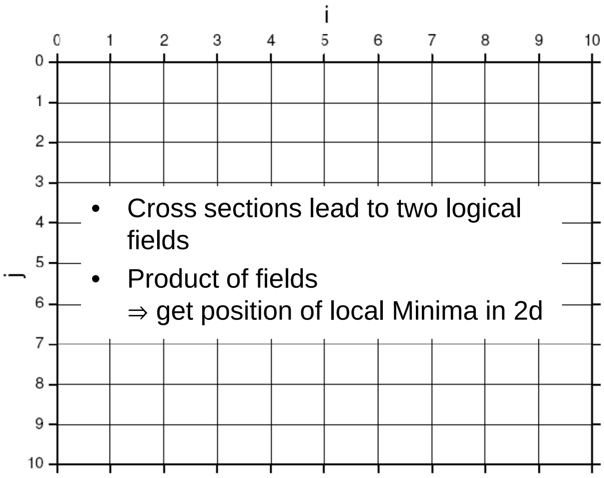
- Comparison with previous and following grid point (descent method)
- For all i=1,...,m and j=1,...,n

1-dimensional

- Question:
  y(x) ≦ y(x-1)
  and
  y(x) ≦ y(x+1)
- Answer:
  yes: index = 1
  no: index = 0
- Result : logical field
- Example:
  x=0,...,20 ; y=sin(x)
  if x=5,11,17: index 1
  else : index 0



2-dimensional



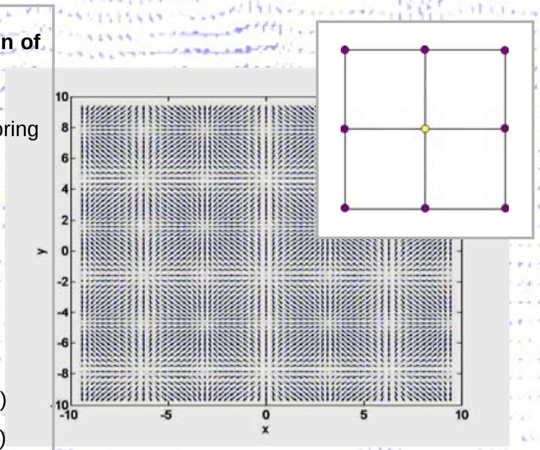
- Cross sections lead to two logical fields
- Product of fields
  ⇒ get position of local Minima in 2d

⇒ Sensitiv to noise and grid resolution

Classification

Classification by determination of vector direction

- consider vector field at neighboring grid points
- direction of vector < directness $\pm 45°$?
  - yes=1
  - no=0
- Sum ⇒ how many 1 ?
  - all ⇒ stable (- 1)
  - non ⇒ unstable (+1)
  - some ⇒ saddle, center (0)



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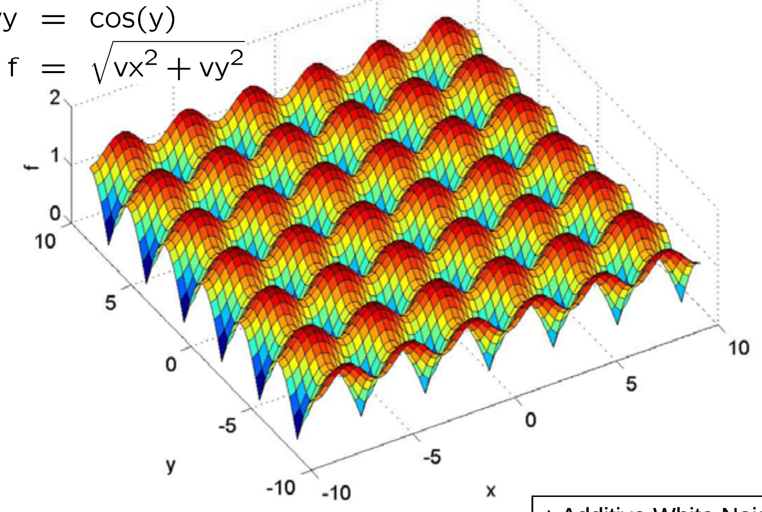
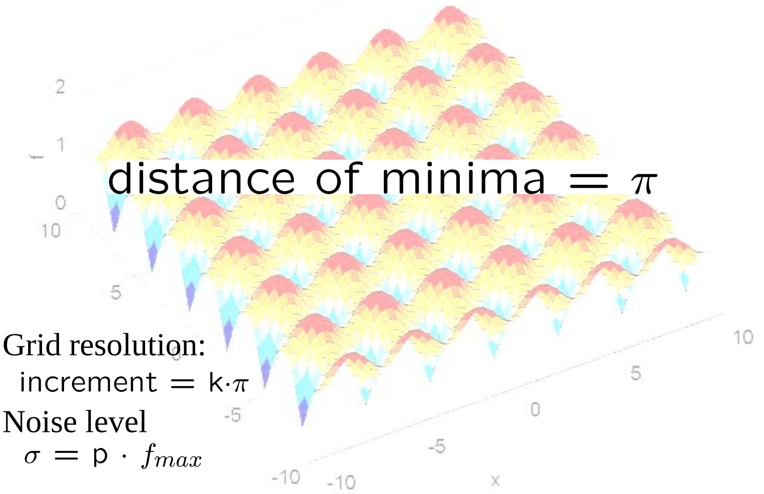
Test function

$$vx = \sin(x)$$
$$vy = \cos(y)$$
$$f = \sqrt{vx^2 + vy^2}$$



+ Additive White Noise

Test function



distance of minima $= \pi$

Grid resolution:

increment $= \mathrm{k} \cdot \pi$

Noise level

$\sigma = \mathrm{p} \cdot f_{max}$

Additions

Smoothing and interpolation

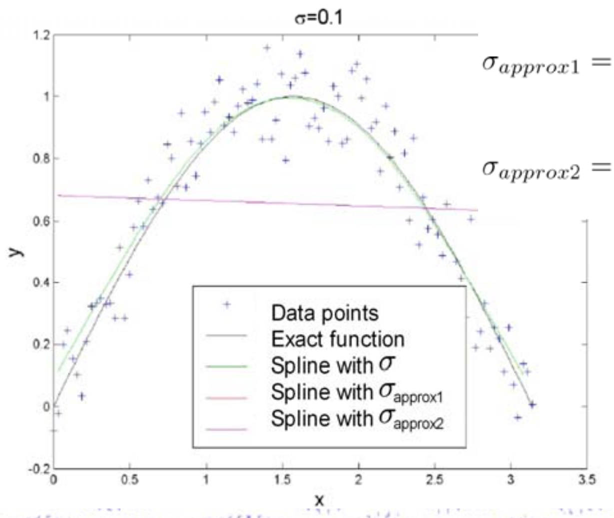
- MA filter
  - Centered window
  - Smoothing parameter b window size: 2(b-1)+1
- 1d spline with smoothing
  - cubic spline with additional minimization of curvature (Reinsch 1967)
- 2d spline interpolation
  - addional points in-between grid (implemented in Matlab)

Further conditions

- Variance
  - consider values < var/b only
  - var: mean variance of increments b: smoothing parameter (MA)
- Height
  - consider values < 5% of mean height of |v| only
- Monotony
  - consider provious and next two points

Method of Reinsch

Christian H. Reinsch. Smoothing by Spline Functions. *Numerische Mathematik 10*, pages 77–83, 1967.

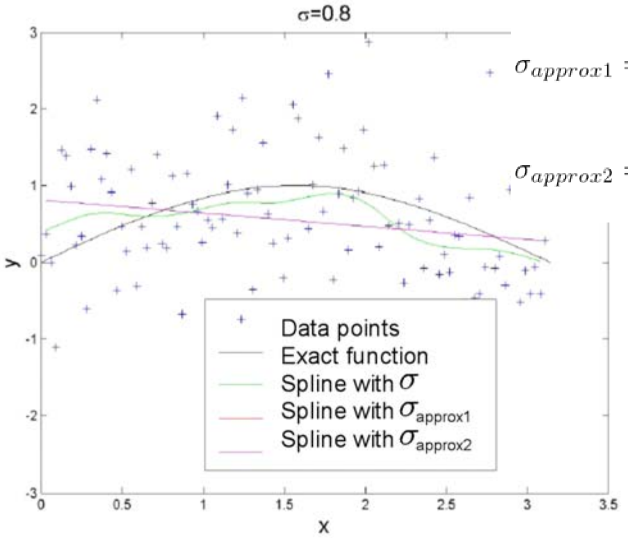


$$\sigma_{approx1} = \left(\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{\frac{1}{2}}$$

$$\sigma_{approx2} = \left(\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{\frac{1}{2}}$$

Very sensitiv to approximation of $\boldsymbol{\sigma}$

Method of Reinsch



$$\sigma_{approx1} = \left(\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{\frac{1}{2}}$$

$$\sigma_{approx2} = \left(\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{\frac{1}{2}}$$

standard deviation $\boldsymbol{\sigma}$ is know exactly
⇒ good results for 1d also for high noise level

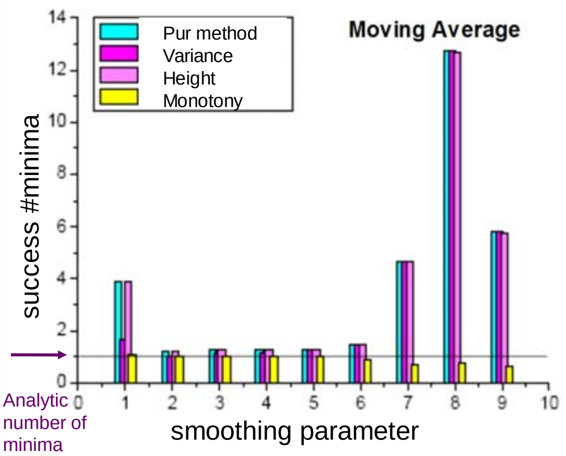
Results for positions

spline with smoothing

- If noise level > 30% to much smoothing ⇒ no minima

spline interpolation

- Minima also for higher noise level (> 100%)



Increments of spline grid so that analytical minima at grid points

Increment 0.1
Standard deviation of noise 0.05

Results for positions

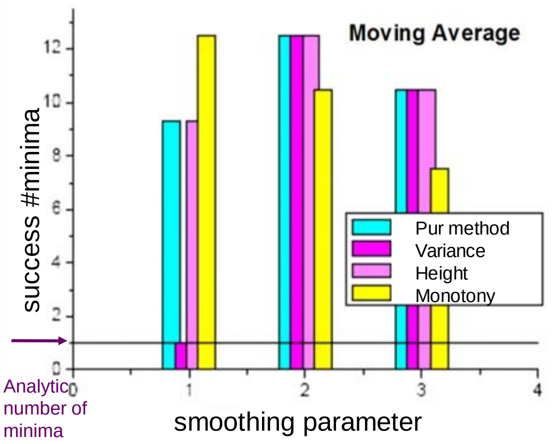
spline with smoothing

- If noise level > 30% to much smoothing ⇒ no minima

spline interpolation

- Minima also for higher noise level (> 100%)

Increments of spline grid so that analytical minima at grid points



Increment 0.3
Standard deviation of noise 0

Results for classification

- If position of fixed points is exactly know classification by direction of vectors is successful
- Not very senitiv to noise

Bower's model

- Model of meandering jet stream
- Stream function

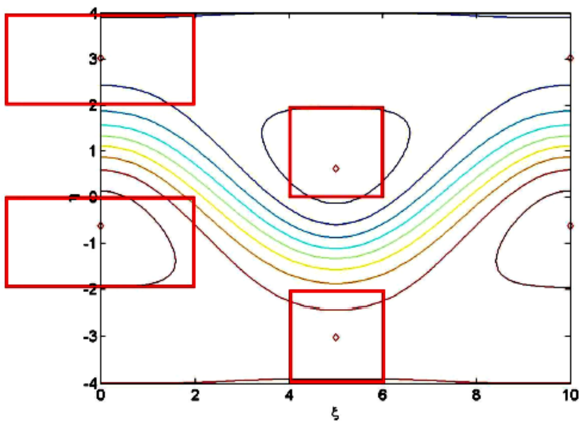
$$\psi(x,y,t) = \psi_0 \left[1 - \tanh\left(\frac{y - A\cos(k(x - c_x t))}{\sqrt{1 + k^2 A^2 \sin^2(k(x - c_x t))}}\right)\right]$$

nondimensional

$$\phi(\xi,\eta) \quad = \quad 1 - \tanh\left(\frac{\eta - B\cos(\kappa\xi)}{\sqrt{1 + \kappa^2 B^2 \sin^2(\kappa\xi)}}\right) + c\eta$$

Stream function

Parameter:
B=1.2
L=$2\pi\kappa^{-1} = 10$
c=0.1
increment=0.1



Fixed points analytical computable

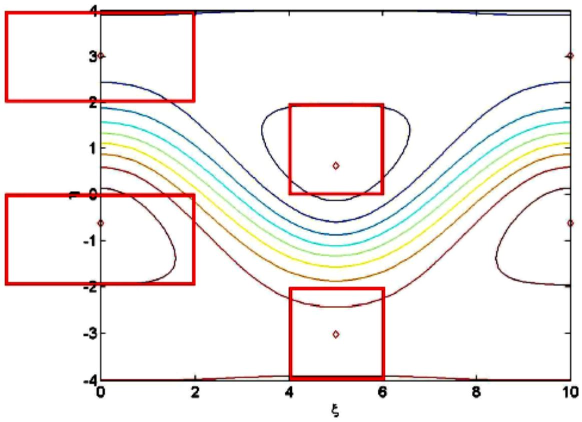
hyperbolic

$\xi_k = kL,\ \eta_k = artanh(\sqrt{1-c}) + B,\ k \in \mathbb{Z}$

$\xi_k = \frac{L}{2} + kL,\ \eta_k = -artanh(\sqrt{1-c}) - B,\ k \in \mathbb{Z}$

Stream function

Parameter:
B=1.2
L=2$\pi\kappa^{-1} = 10$
c=0.1
increment=0.1



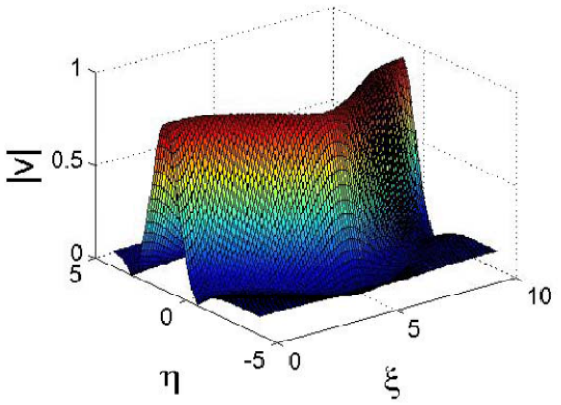
Fixed points analytical computable

elliptic

$$\xi_k = kL,\ \eta_k = -artanh(\sqrt{1-c}) + B,\ k \in \mathbb{Z}$$

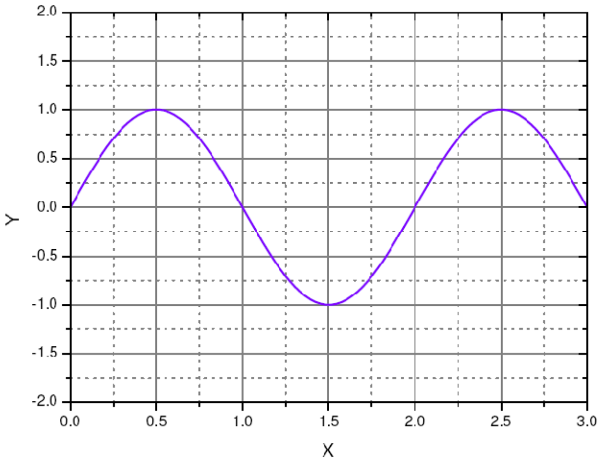
$$\xi_k = \frac{L}{2} + kL,\ \eta_k = artanh(\sqrt{1-c}) - B,\ k \in \mathbb{Z}$$

Problem



- Minima difficult to detect because of flatness
- needs high resolution ⇒ only small areas (capacity of matlab)

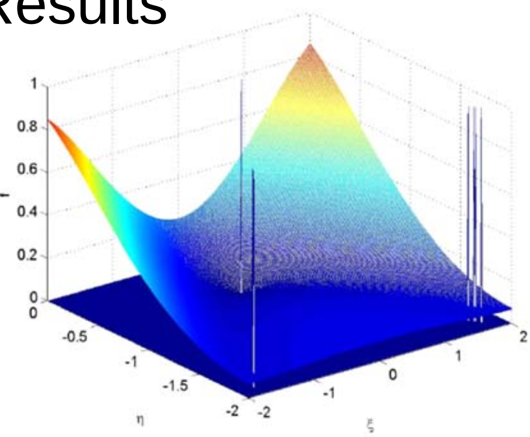
Problem

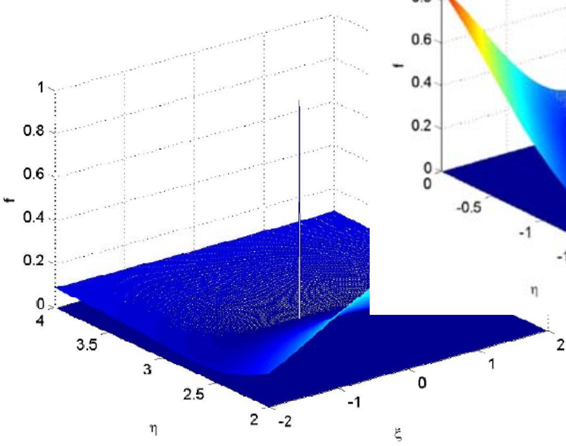


Minima very flat when hight propability of getting to much minima (if condition >=)

Rounding errors misrepresent results

Results





For increment 0.01 (no noise) acceptable results (besides 1d spline and fixed height limit)

Application to netcdf data

- No analytic fixed points known
- For evaluation of methodology reference values needed
  - Assumption: no bifurcation
  - Determination of fixed points: all minima found also in previous and next time step
  - Condition: fixed points can move by on cell per time step only

Method

- Determination of fixed points using previous and next time step
- Comparison with results of earlier used method (variable hight limit)

- Hight limit as ratio of moments certain order
- use central moments and mean values

$$h = \frac{m_i}{m_j}$$

i, j: order of moments

Results

Example: January 1985

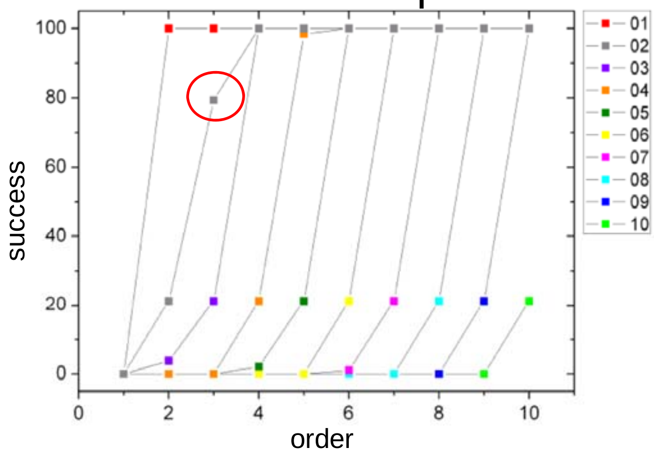
$$h = \frac{m_i}{m_j}$$

where m

- if i=1 or j=1 mean value
- if i>1 or j>1 central moments of order 2 to 10

Abscissa: m_i
Different colors: m_j

identified fixed points



Results

Example: January 1985

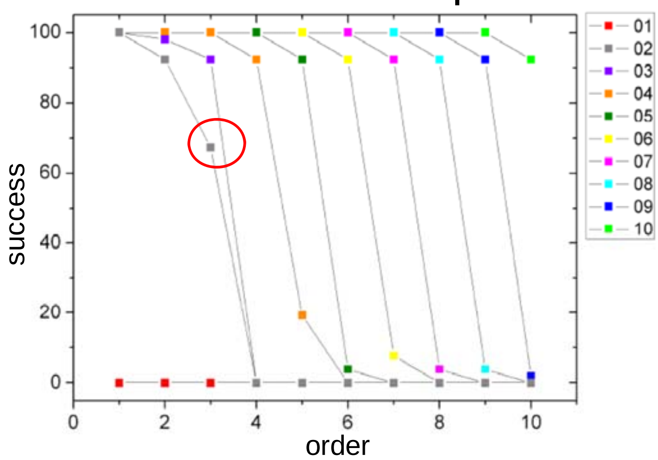
$$h = \frac{m_i}{m_j}$$

where m

- if i=1 or j=1 mean value
- if i>1 or j>1 central moments of order 2 to 10

Abscissa: m_i
Different colors: m_j

identified not fixed points



Limit of height

$$h = \frac{m_i}{m_j}$$

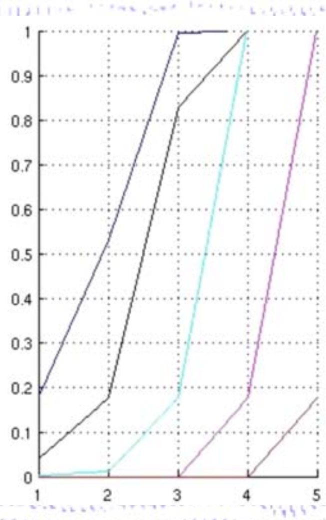
Best results for i=3 and j=2

$E((x-\mu)^3)/E((x-\mu)^2)$
with μ mean

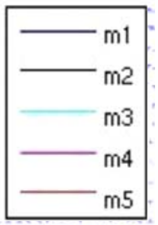
Important skewness and standard deviation

skewness $= \frac{E((x-\mu)^3)}{\sigma^3}$
standard deviation $= \sigma$

Influence of chemistry

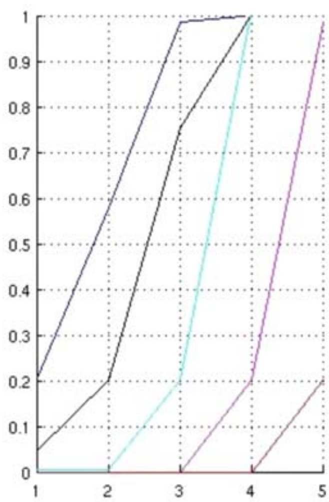


moment



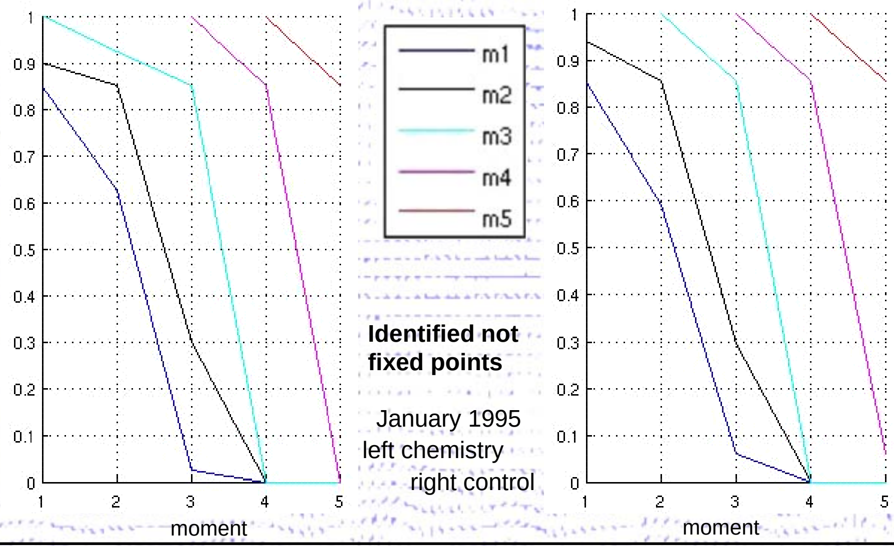
Identified fixed points

January 1995
left chemistry
right control



moment

Influence of chemistry



Identified not fixed points

January 1995
left chemistry
right control

Summary

- Determination of local minima in velocity fields to find fixed points very senitiv to
  - Resolution of grid
  - Noise
- Improvement through interpolation and additional limiting conditions possible
  - Interpolated grid must fit
  - Minima not too flat
  - Proper limits depending on moments
- Lightly different results concerning fixed points for netcdf data with and without stratospheric chemistry feedback