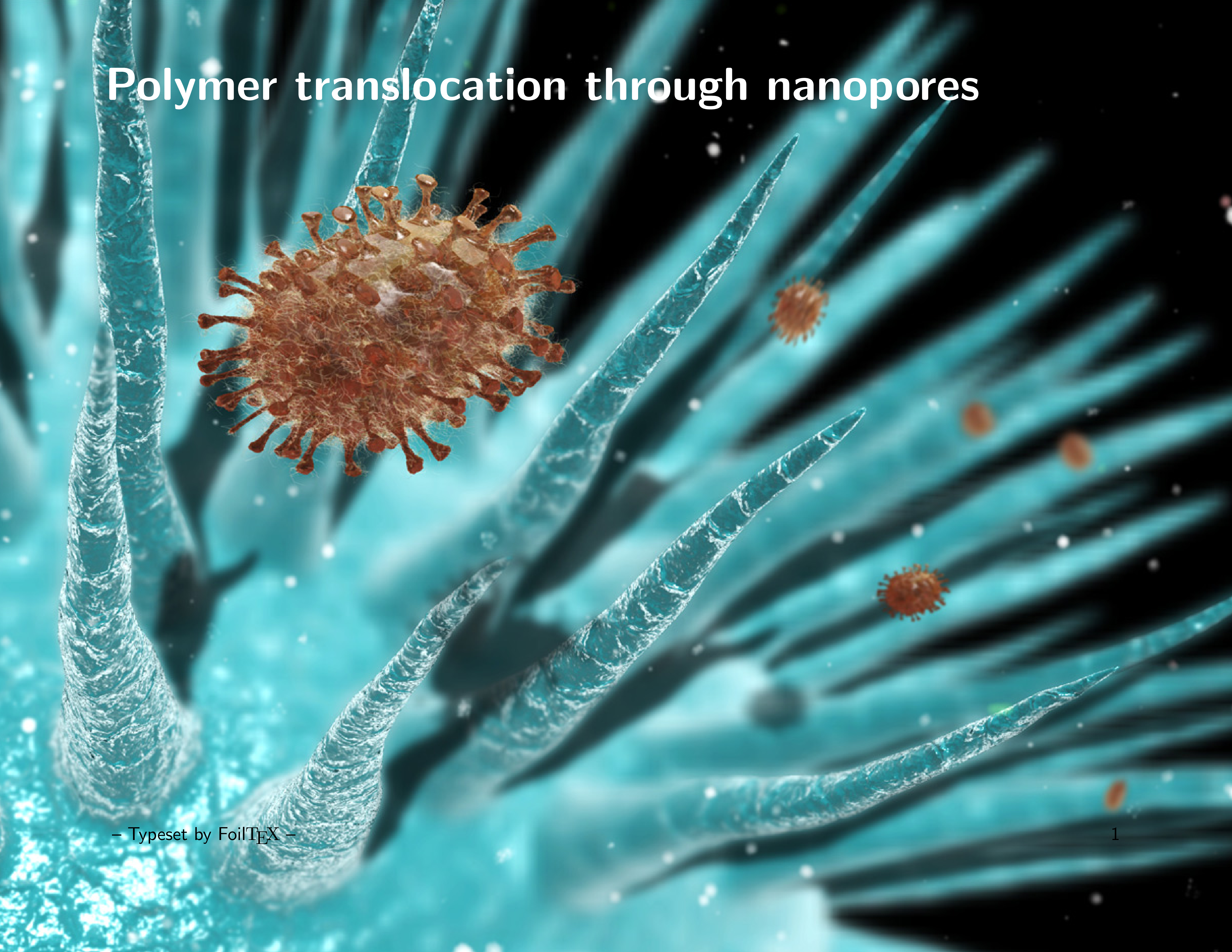
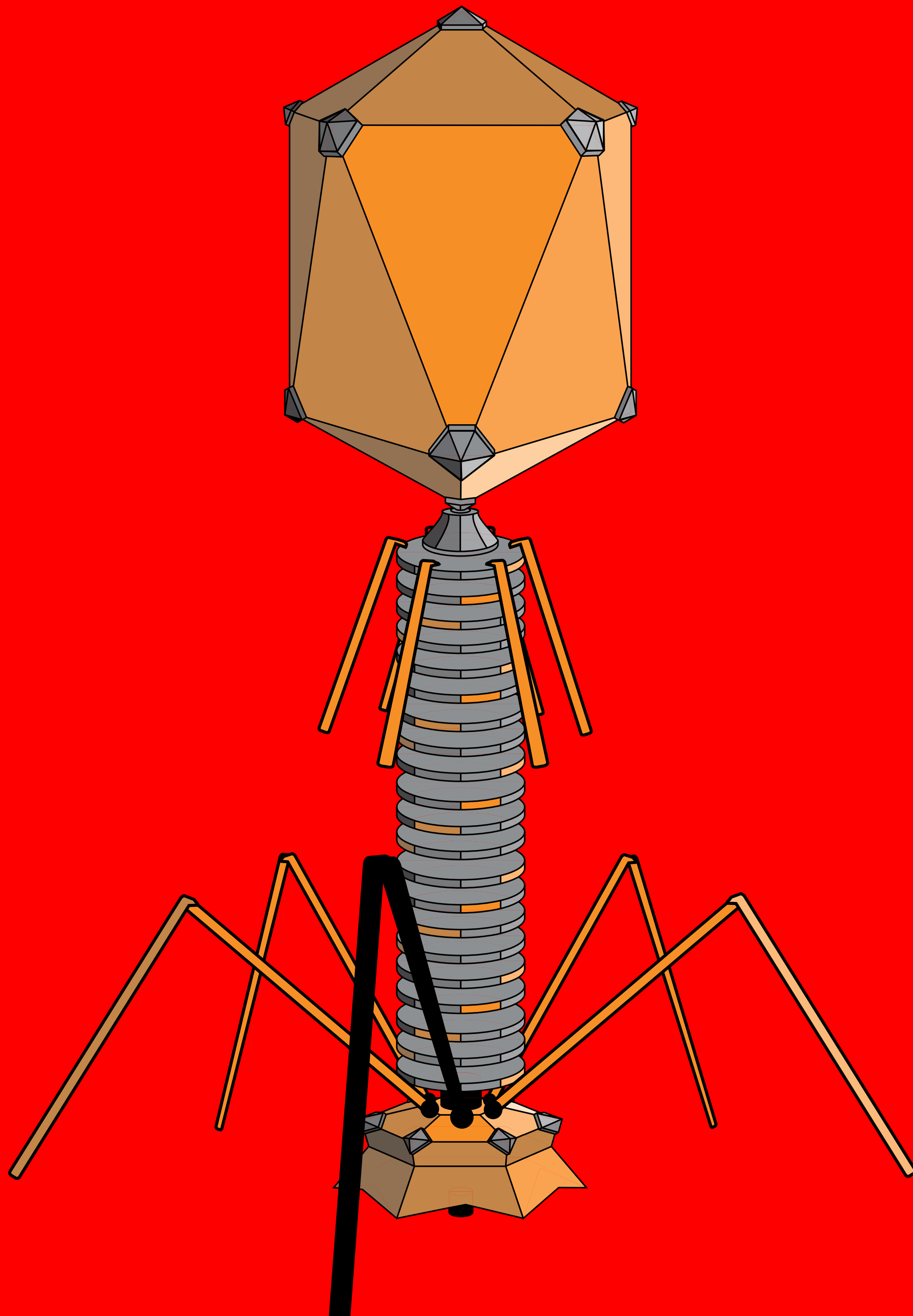
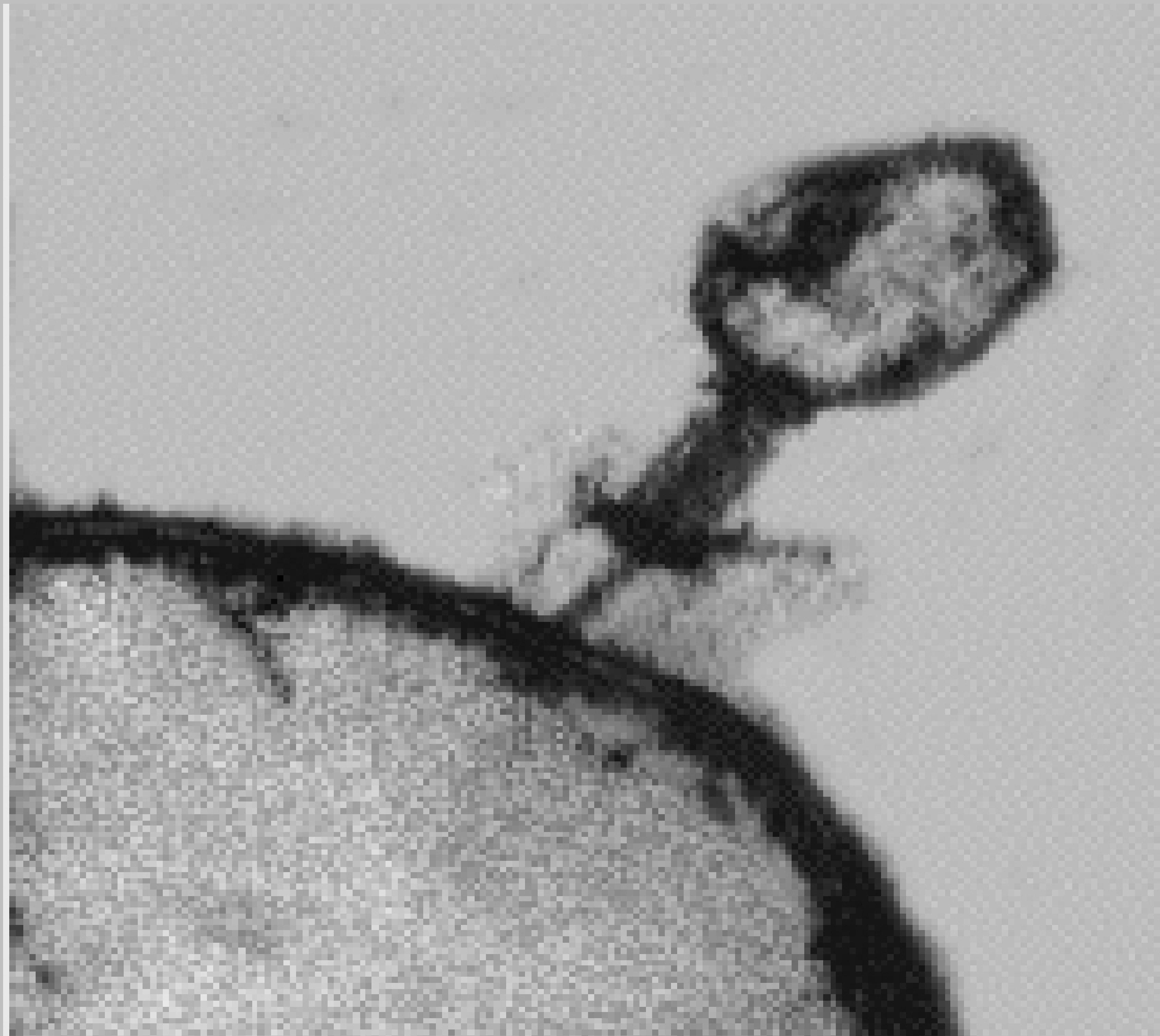
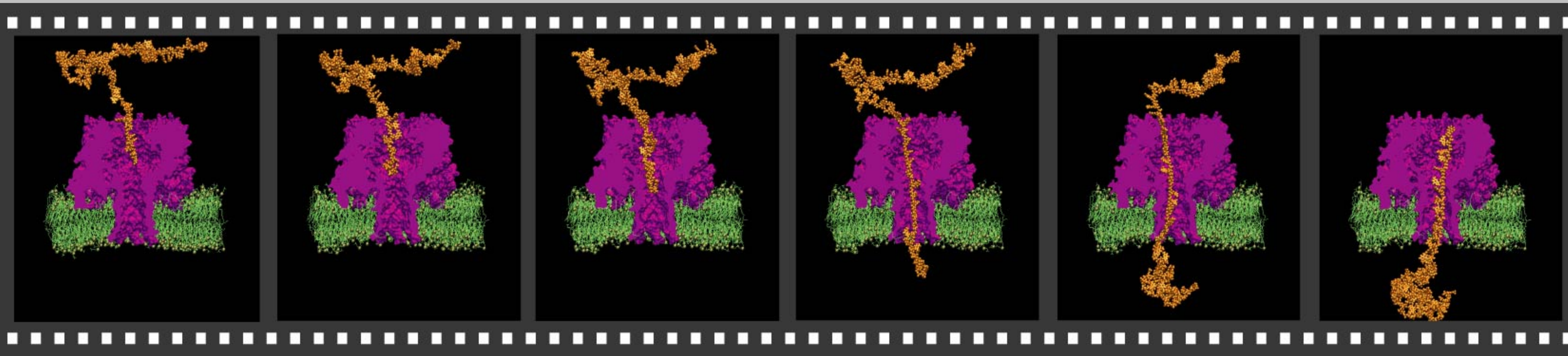
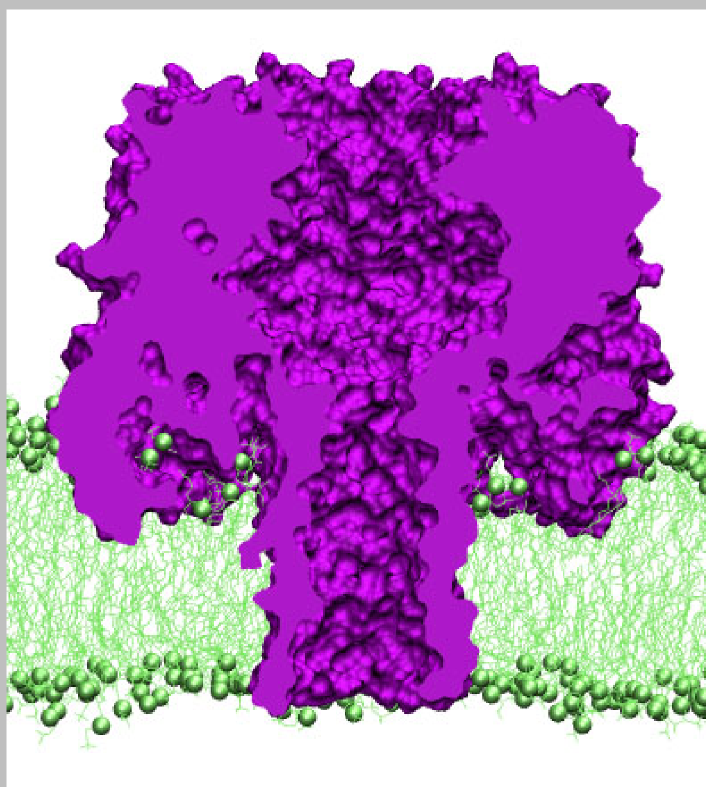
Polymer translocation through nanopores

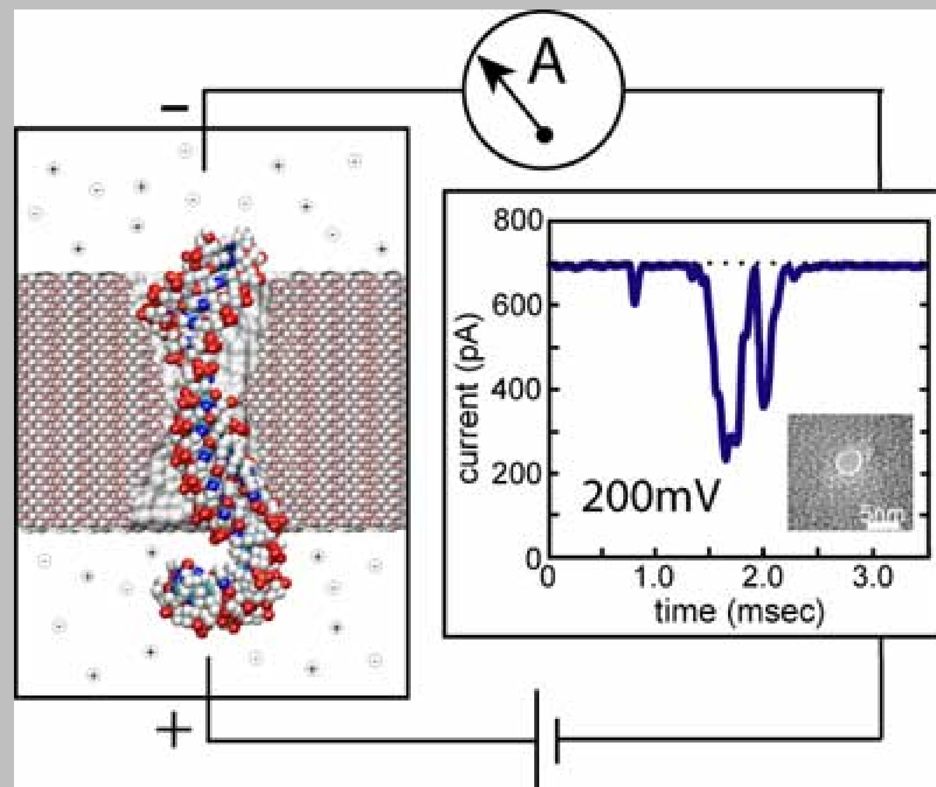


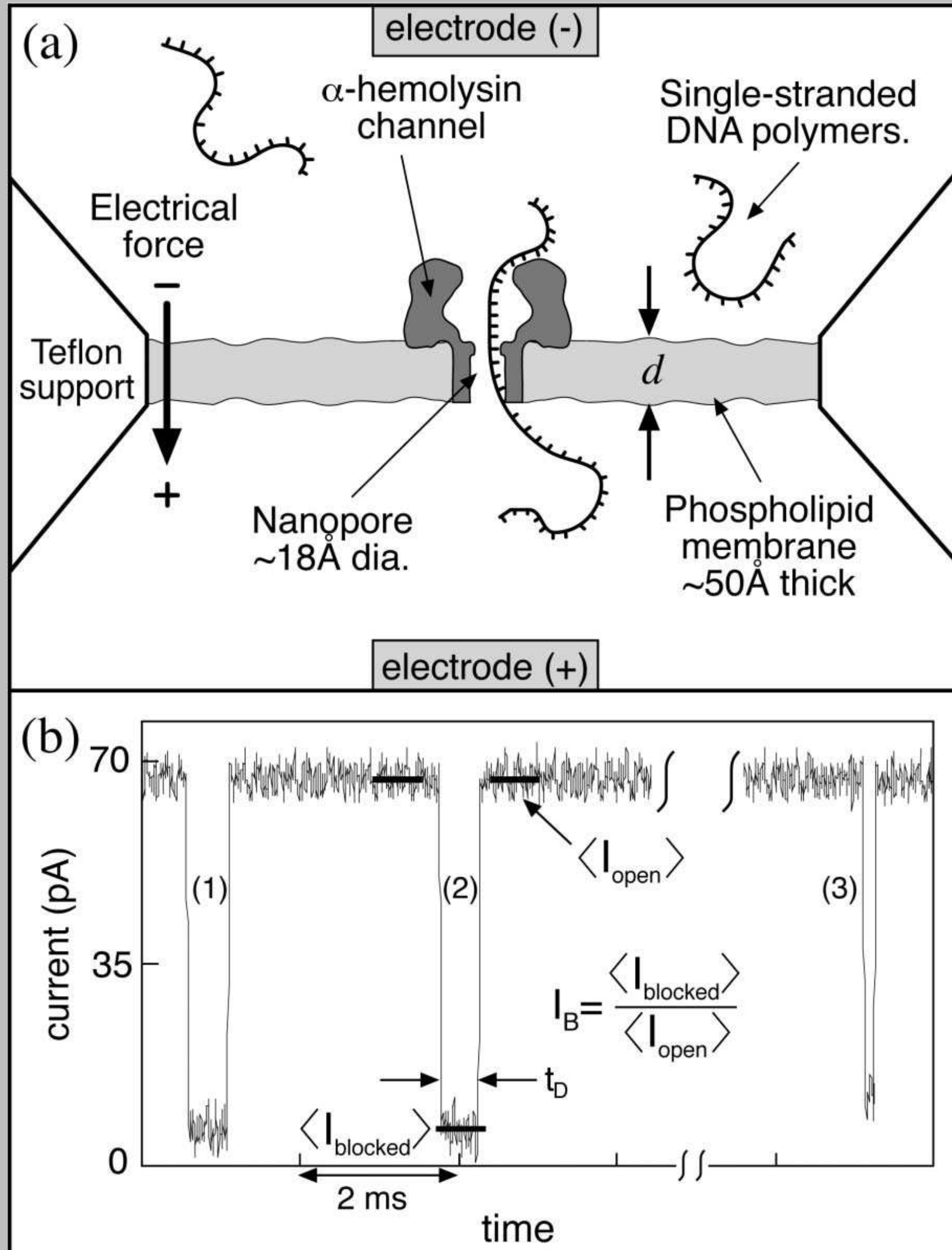




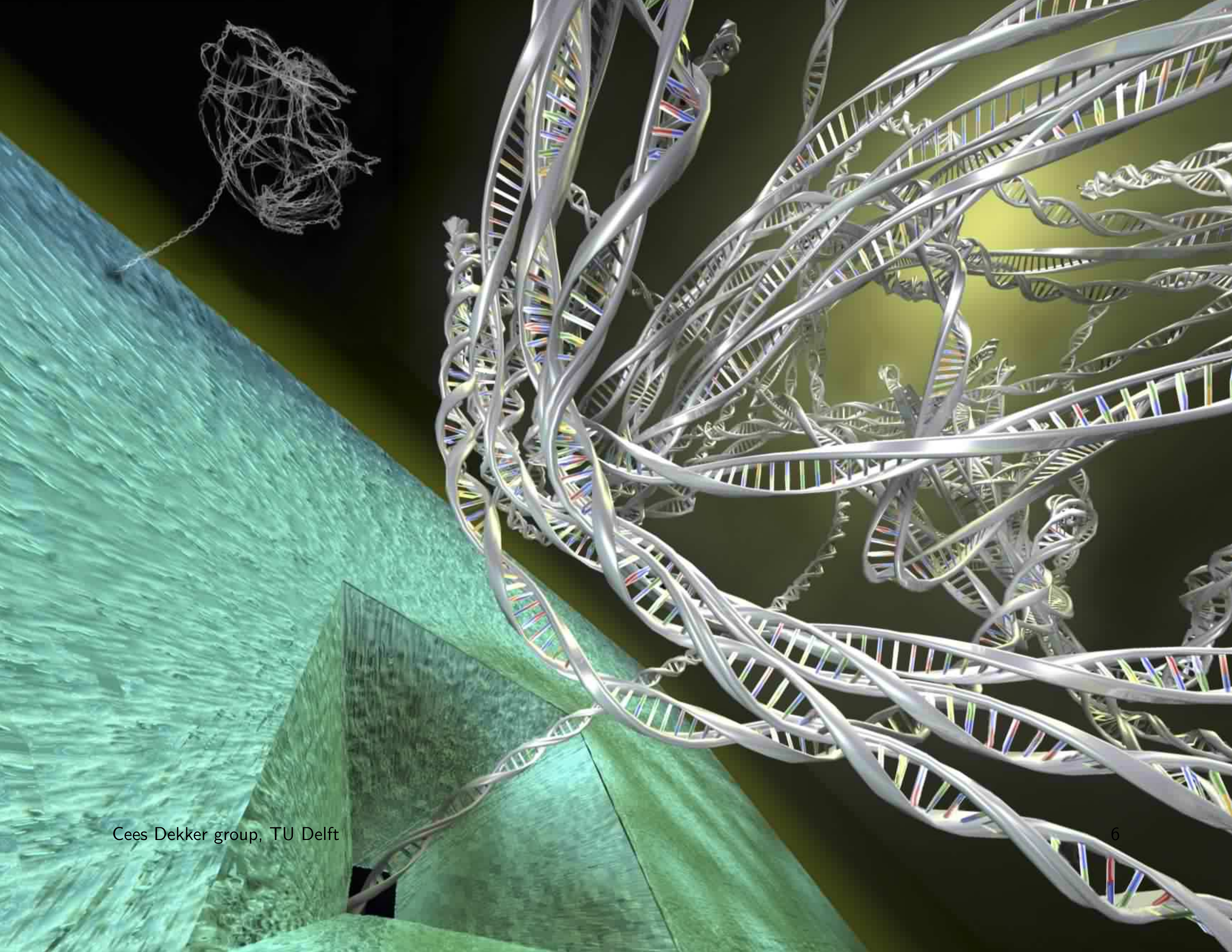





A
−
+
800
600
400
200
0
0
1.0
2.0
3.0
current (pA)
time (msec)
200mV



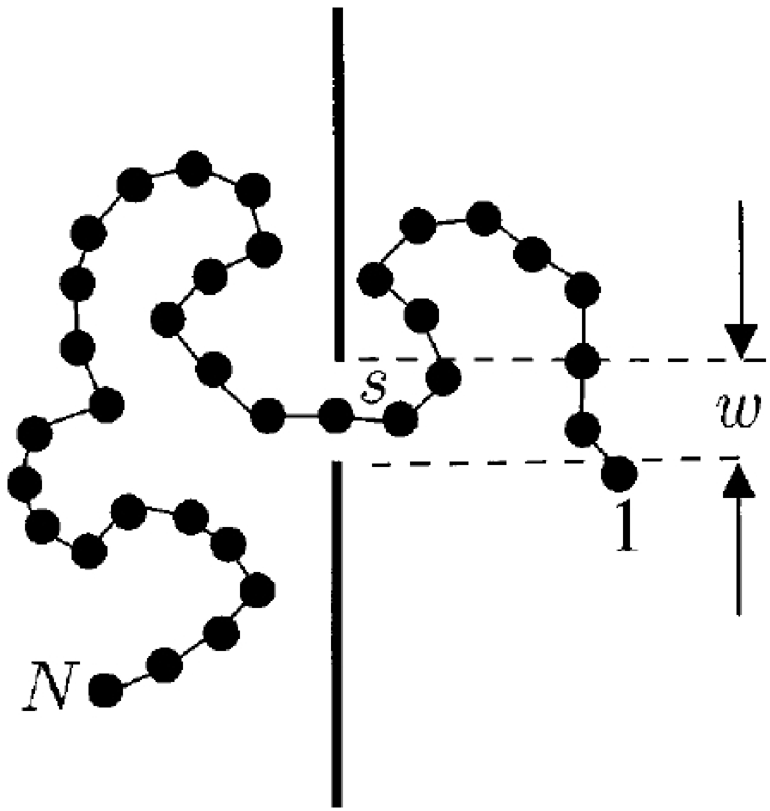
A Meller, L Nivon & D Brenton, PRL 86 (2001)



Polymer model for translocation

1. Chain is polymer with N monomers
2. Chain is already threaded into pore
3. Reflecting B.C. @ $s = 0$
4. Absorbing B.C. @ $s = N$
5. Neglect chain-pore interactions
6. $\exists$ reaction coordinate:

$$s = \# \left\{ \begin{array}{c} \text{translocated} \\ \text{monomers} \end{array} \right\}$$



J Chuang, Y Kantor & M Kardar, PRE (2001)

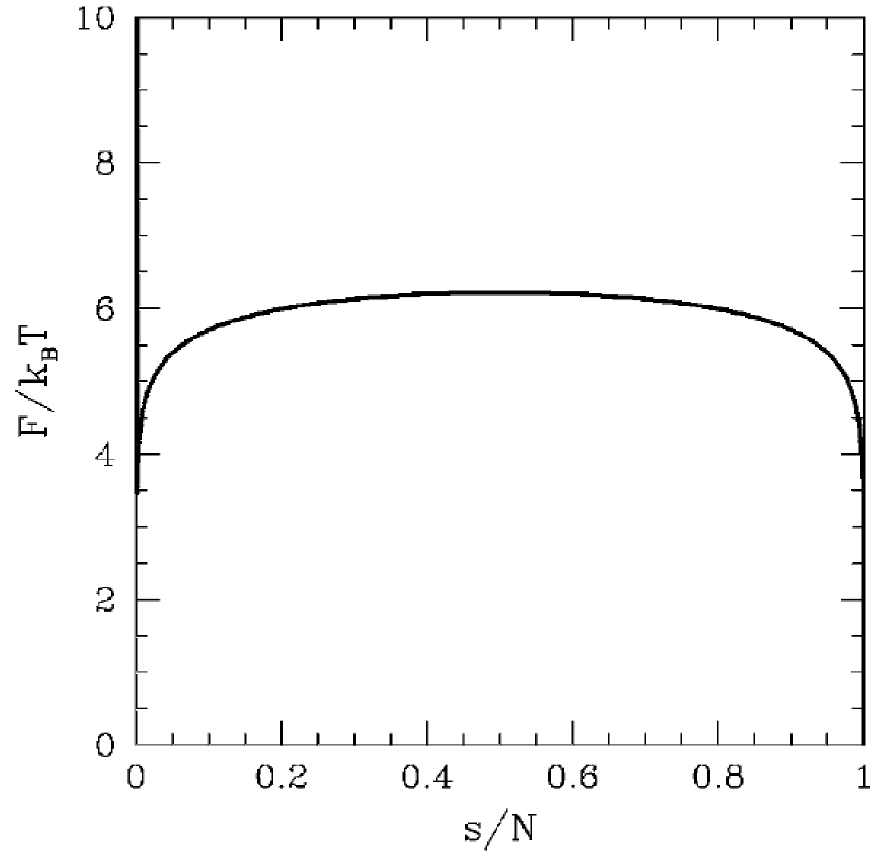
Entropic barrier

d.o.f. of a chain with N monomers:

$$\omega \simeq \mu^N N^{\gamma_{\mathscr{G}}-1}$$

s-dependence of free energy

$$\mathscr{F}(s) \simeq (\gamma_{\mathscr{G}} - 1) k_B T \log \Big([N - s] s \Big)$$



$\rightsquigarrow$ driving force needed for efficient translocation

J Chuang, Y Kantor & M Kardar, PRE (2001)

Driving force

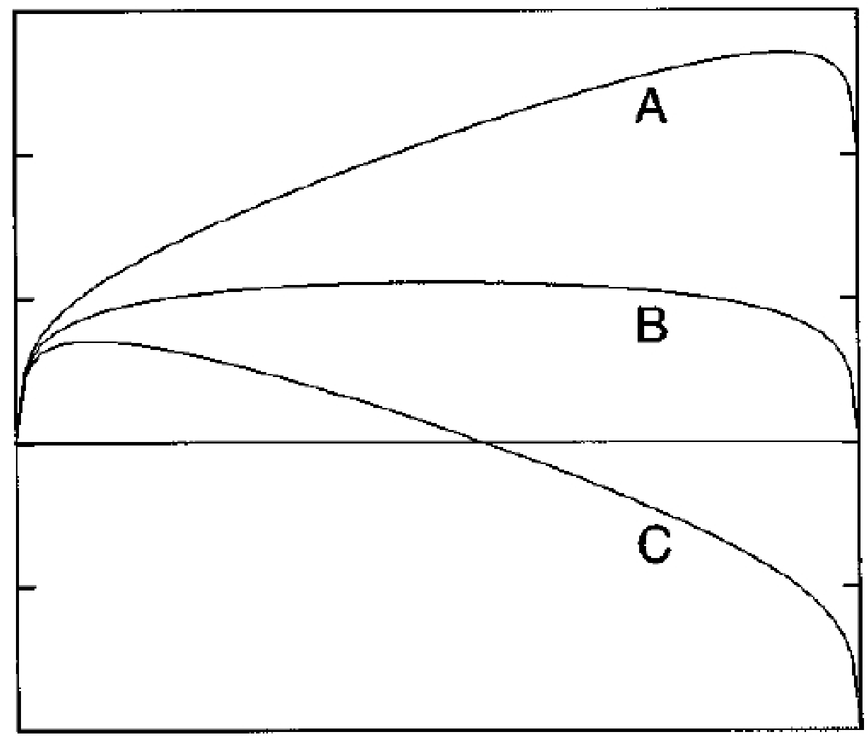
Driving force creates chemical potential difference $\Delta\mu$ per monomer $\rightsquigarrow$ drift

1. Trans-membrane potential

2. Binding proteins

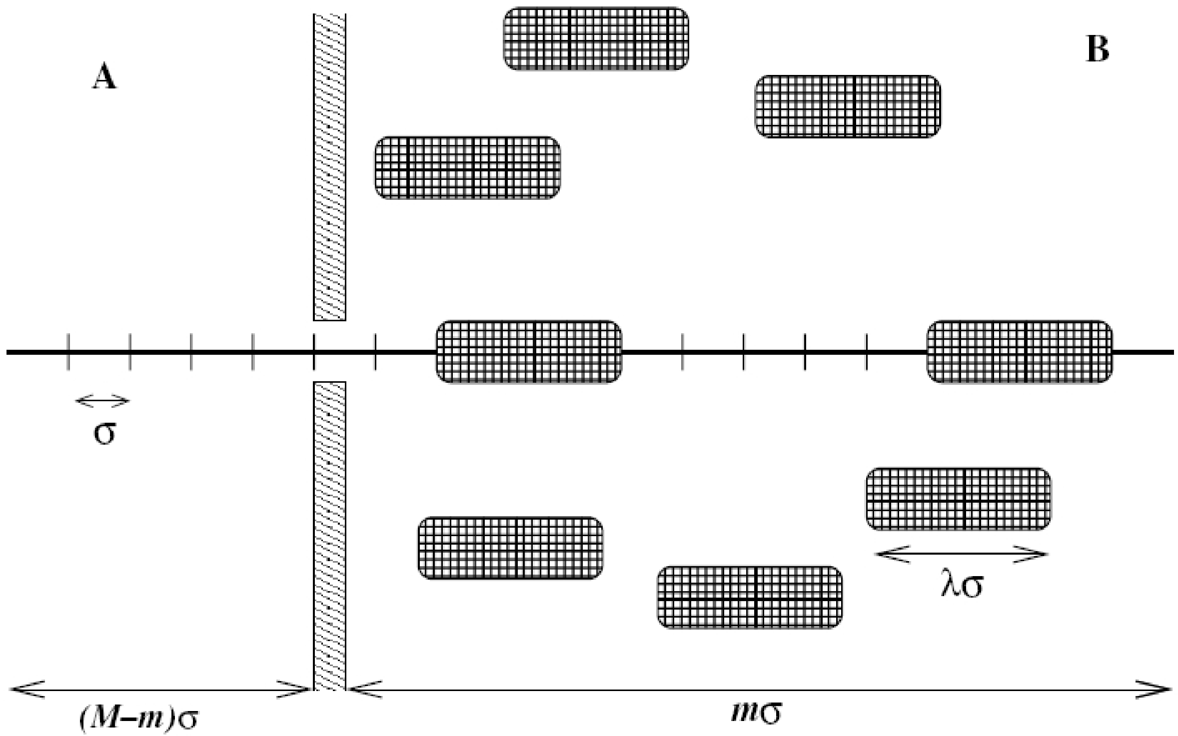
3. Cis confinement (e.g., virus)

4. Active pulling



Sung & Park, PRL 77 (1996)

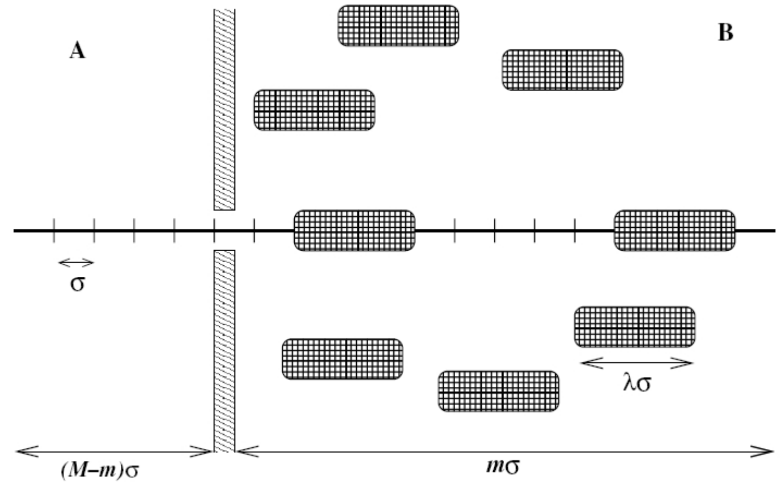
Chaperone-driven translocation



T Ambjörnsson & RM, Physical Biology (2004); T Ambjörnsson, MA Lomholt & RM, J Phys Cond Mat (2005)

Chaperone-driven translocation

Driving: neglect polymeric d.o.f.

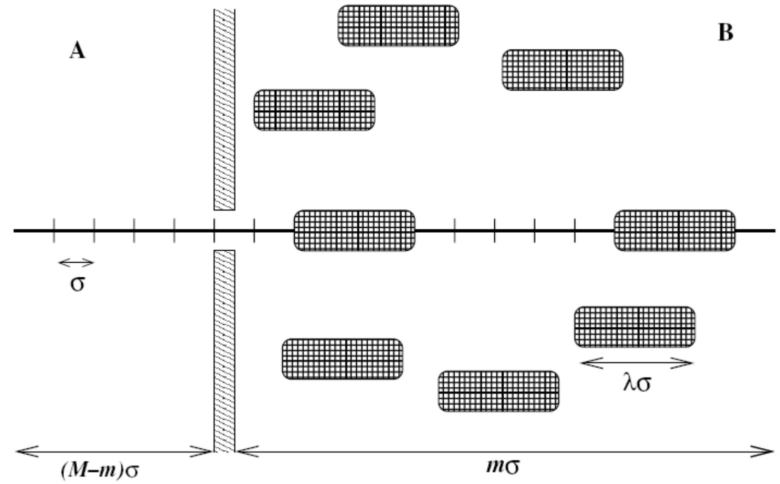


Chaperone-driven translocation

Driving: neglect polymeric d.o.f.

Forward translocation rate:

$$t^{+}(m,n) = k$$



Chaperone-driven translocation

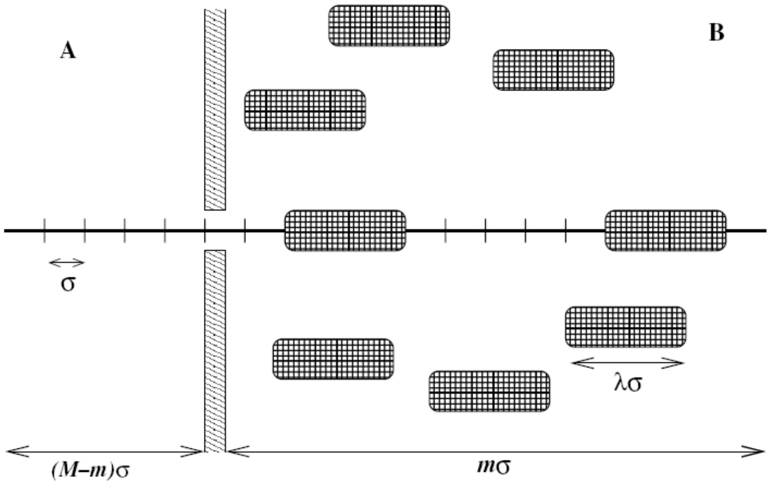
Driving: neglect polymeric d.o.f.

Forward translocation rate:

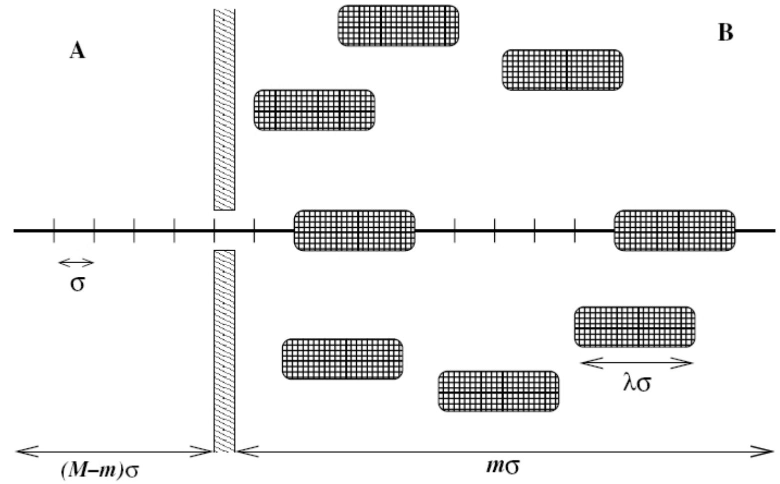
$$\mathrm{t}^{+}(m,n) = k$$

Backward translocation rate:

$$\mathrm{t}^{-}(m,n) = k \times \mathrm{Pr}\left\{\begin{array}{c}\text{trans binding site}\\ \text{site closest to}\\ \text{pore is vacant}\end{array}\right\}$$



Chaperone-driven translocation



Driving: neglect polymeric d.o.f.

Forward translocation rate:

$$\mathrm{t}^{+}(m,n) = k$$

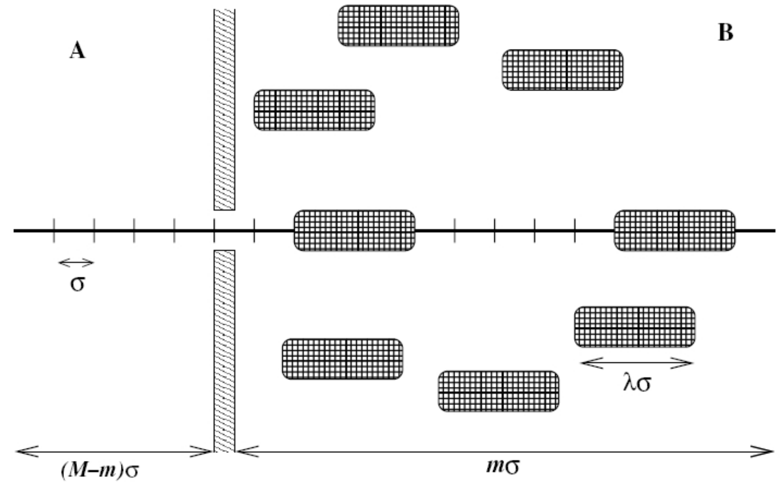
Backward translocation rate:

$$\mathrm{t}^{-}(m,n) = k \times \mathrm{Pr}\left\{ \begin{array}{c} \text{trans binding site} \\ \text{site closest to} \\ \text{pore is vacant} \end{array} \right\}$$

Chaperone unbinding rate:

$$\mathrm{r}^{-}(m,n) = nq$$

Chaperone-driven translocation



Driving: neglect polymeric d.o.f.

Forward translocation rate:

$$\mathrm{t}^{+}(m,n) = k$$

Backward translocation rate:

$$\mathrm{t}^{-}(m,n) = k \times \mathrm{Pr}\left\{\begin{array}{c}\text{trans binding site}\\ \text{site closest to}\\ \text{pore is vacant}\end{array}\right\}$$

Chaperone unbinding rate:

$$\mathrm{r}^{-}(m,n) = nq$$

Chaperone binding rate:

$$\mathrm{r}^{+}(m,n) = c_0 K^{\mathrm{eq}} \times \mathscr{N}\left\{\begin{array}{c}\text{ways to add addtl}\\ \text{chaperone if}\\ n \text{ bound already}\end{array}\right\}$$

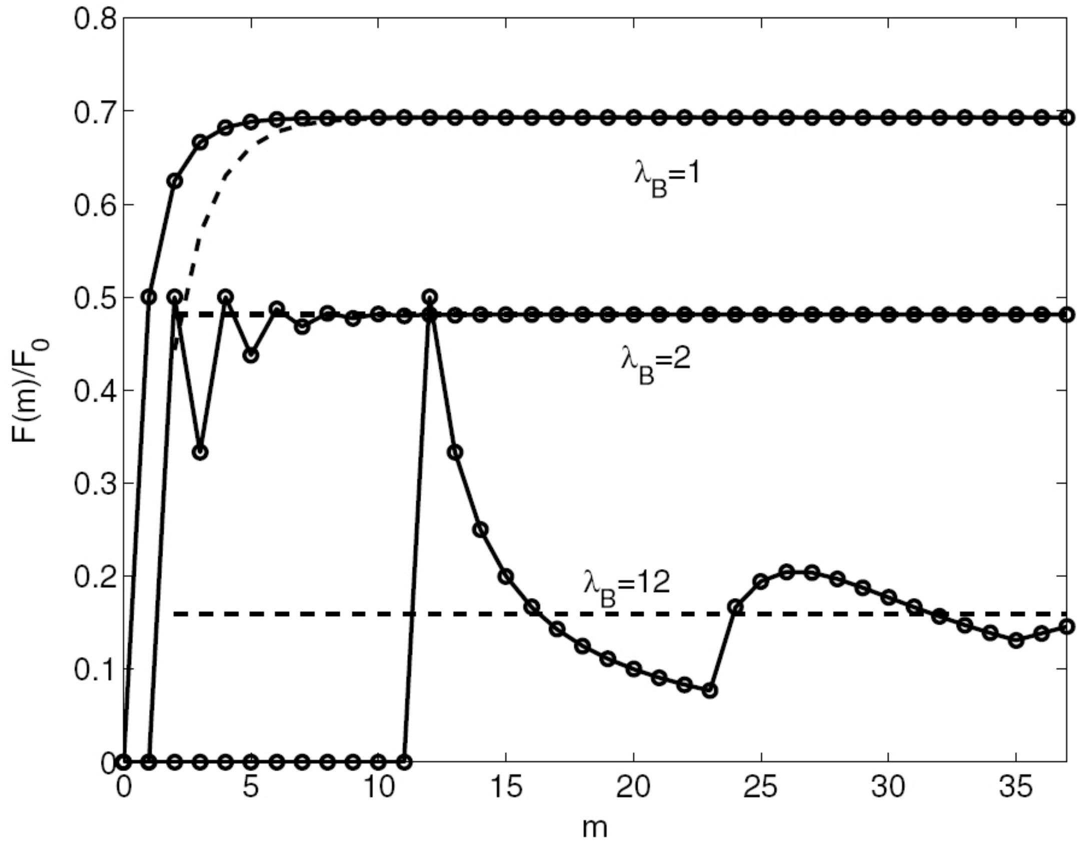
T Ambjörnsson & RM, Physical Biology (2004); T Ambjörnsson, MA Lomholt & RM, J Phys Cond Mat (2005)

Chaperone-driven translocation

1. Slow binding dynamics: purely diffusive motion $\tau_T \simeq N^2$
2. Slow unbinding dynamics: ratcheted motion $\tau_T \simeq N$
3. Fast binding/unbinding: adiabatic elimination of chaperone dynamics

Chaperone-driven translocation

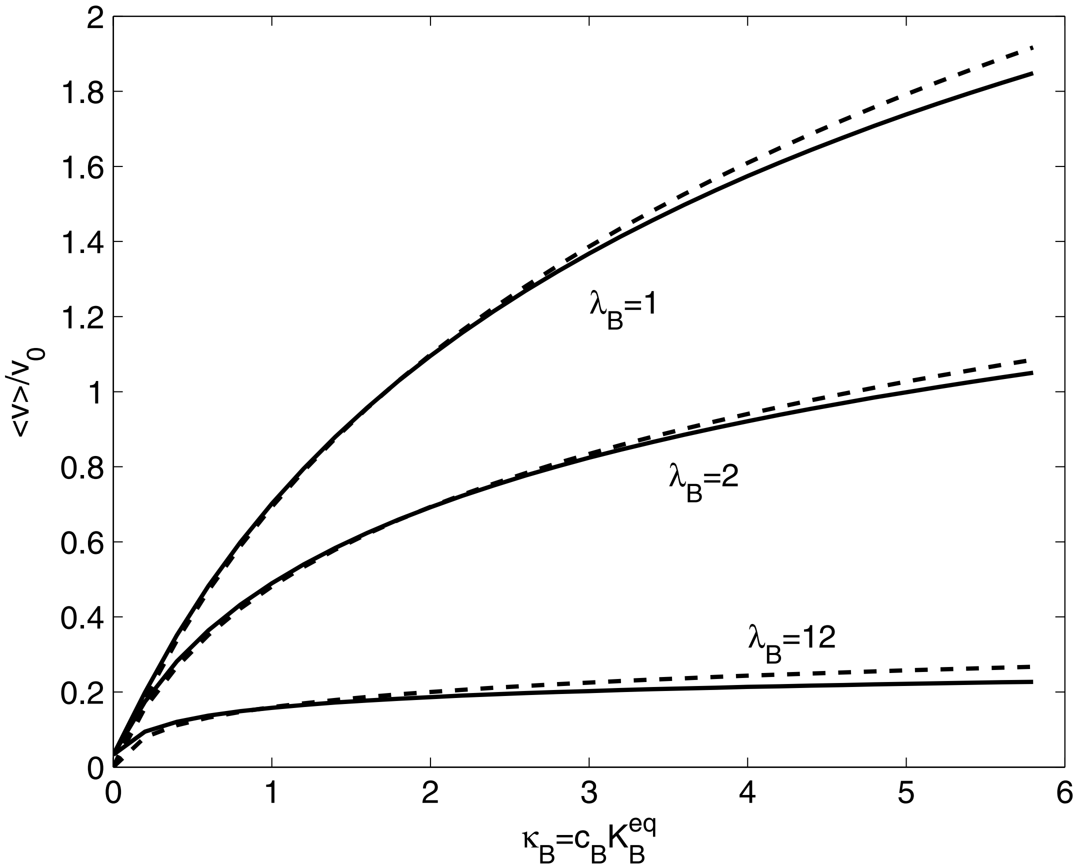
1. Slow binding dynamics: purely diffusive motion $\tau_T \simeq N^2$
2. Slow unbinding dynamics: ratcheted motion $\tau_T \simeq N$
3. Fast binding/unbinding: adiabatic elimination of chaperone dynamics



T Ambjörnsson & RM, Physical Biology (2004); T Ambjörnsson, MA Lomholt & RM, J Phys Cond Mat (2005)

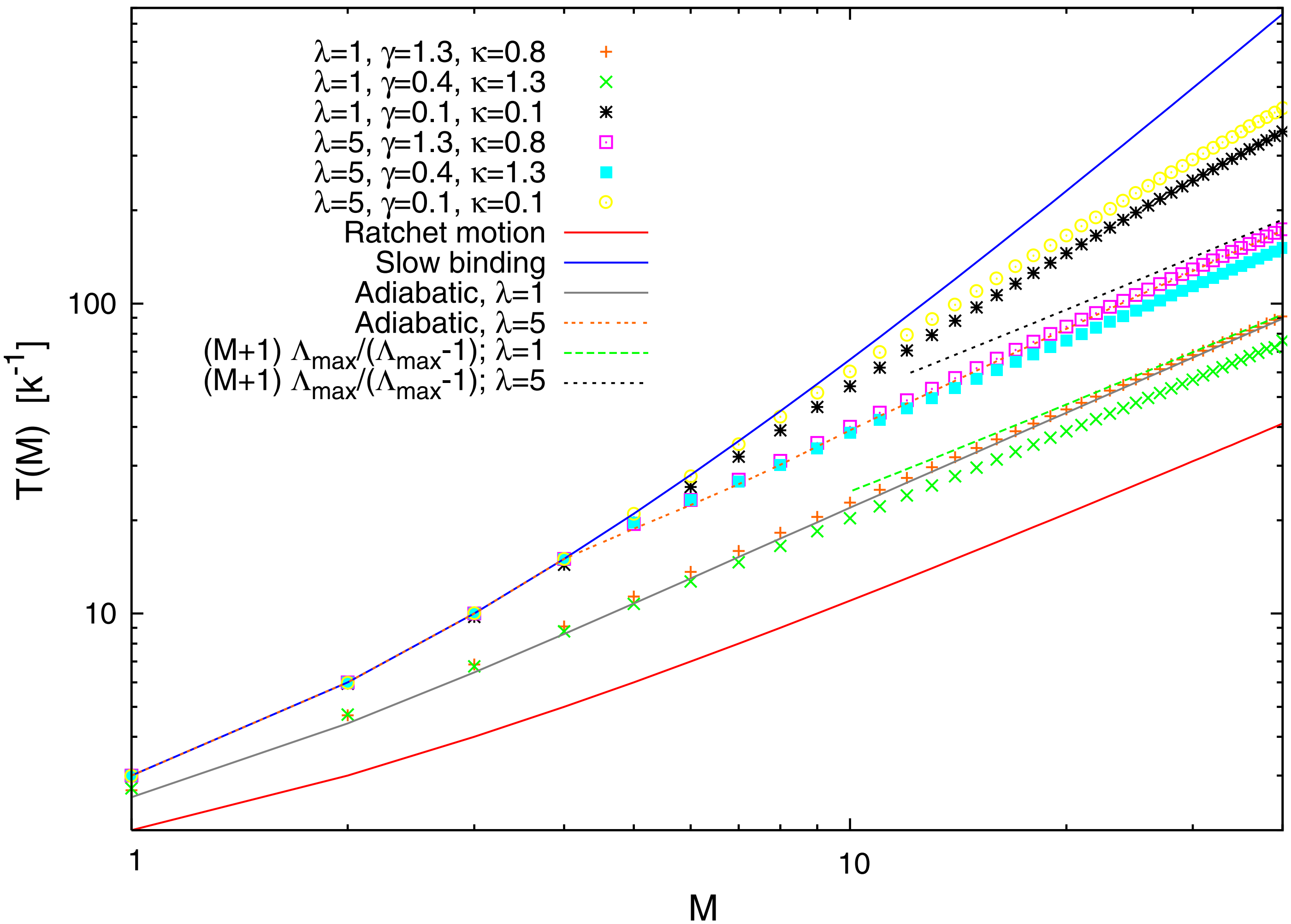
Chaperone-driven translocation

Dependence of mean translocation velocity on binding strength:



Chaperone-driven translocation

Mean translocation time as function of chain length:



T Ambjörnsson & RM, Physical Biology (2004); T Ambjörnsson, MA Lomholt & RM, J Phys Cond Mat (2005)

Polymer translocation: many groups, many opinions ...

Brownian approach to translocation

Translocation dynamics sufficiently slow $\rightsquigarrow$ diffusion in potential with PDF $P(s,t)$

Brownian approach to translocation

Translocation dynamics sufficiently slow $\rightsquigarrow$ diffusion in potential with PDF $P(s,t)$

Continuity equation:

$$\frac{\partial}{\partial t}P(s,t) = -\frac{\partial}{\partial s}j(s,t)$$

Brownian approach to translocation

Translocation dynamics sufficiently slow $\rightsquigarrow$ diffusion in potential with PDF $P(s,t)$

Continuity equation:

$$\frac{\partial}{\partial t}P(s,t) = -\frac{\partial}{\partial s}j(s,t)$$

Probability current:

$$j(s,t) = -D\left(\frac{\partial}{\partial s}P(s,t) + \frac{P(s,t)}{k_B T}\frac{\partial}{\partial s}\mathscr{F}(s)\right) \quad \therefore \quad D = \frac{\mu}{k_B T}$$

Brownian approach to translocation

Translocation dynamics sufficiently slow $\rightsquigarrow$ diffusion in potential with PDF $P(s,t)$

Continuity equation:

$$\frac{\partial}{\partial t}P(s,t) = -\frac{\partial}{\partial s}j(s,t)$$

Probability current:

$$j(s,t) = -D\left(\frac{\partial}{\partial s}P(s,t) + \frac{P(s,t)}{k_B T}\frac{\partial}{\partial s}\mathscr{F}(s)\right) \quad \therefore \quad D = \frac{\mu}{k_B T}$$

Resulting Fokker-Planck equation (rescaled à la $s \to sN$ & $t \to tD/N^2$):

$$\frac{\partial}{\partial t}P(s,t) = \frac{\partial^2}{\partial s^2}P(s,t) + (\gamma_{\mathscr{G}} - 1)\frac{\partial}{\partial s}\frac{1-2s}{(1-s)s}P(s,t)$$

Brownian approach to translocation

Translocation dynamics sufficiently slow $\rightsquigarrow$ diffusion in potential with PDF $P(s,t)$

Continuity equation:

$$\frac{\partial}{\partial t}P(s,t) = -\frac{\partial}{\partial s}j(s,t)$$

Probability current:

$$j(s,t) = -D\left(\frac{\partial}{\partial s}P(s,t) + \frac{P(s,t)}{k_BT}\frac{\partial}{\partial s}\mathscr{F}(s)\right) \quad \therefore \quad D = \frac{\mu}{k_BT}$$

Resulting Fokker-Planck equation (rescaled à la $s \to sN$ & $t \to tD/N^2$):

$$\frac{\partial}{\partial t}P(s,t) = \frac{\partial^2}{\partial s^2}P(s,t) + (\gamma_{\mathscr{G}} - 1)\frac{\partial}{\partial s}\frac{1-2s}{(1-s)s}P(s,t)$$

Resulting scaling of translocation time $\tau_T \simeq N^2/D$

Brownian approach to translocation

Translocation dynamics sufficiently slow $\rightsquigarrow$ diffusion in potential with PDF $P(s,t)$

Continuity equation:

$$\frac{\partial}{\partial t}P(s,t) = -\frac{\partial}{\partial s}j(s,t)$$

Probability current:

$$j(s,t) = -D\left(\frac{\partial}{\partial s}P(s,t) + \frac{P(s,t)}{k_B T}\frac{\partial}{\partial s}\mathscr{F}(s)\right) \quad \therefore \quad D = \frac{\mu}{k_B T}$$

Resulting Fokker-Planck equation (rescaled à la $s \to sN$ & $t \to tD/N^2$):

$$\frac{\partial}{\partial t}P(s,t) = \frac{\partial^2}{\partial s^2}P(s,t) + (\gamma_{\mathscr{G}} - 1)\frac{\partial}{\partial s}\frac{1-2s}{(1-s)s}P(s,t)$$

Resulting scaling of translocation time $\tau_T \simeq N^2/D$

NB: For s-independent drift: $\tau_T \simeq N/V$ where V the stationary velocity

Sung & Park, PRL 77 (1996), Muthukumar J Chem Phys 111 (1999)

Brownian approach to translocation dynamics?

Kramers escape theory across free energy barrier $\mathscr{F}(s)$ delivers diffusive scaling:

$$\tau_T \simeq \frac{N^2}{D}$$

Brownian approach to translocation dynamics?

Kramers escape theory across free energy barrier $\mathscr{F}(s)$ delivers diffusive scaling:

$$\tau_T \simeq \frac{N^2}{D}$$

First remark: τ_T for crossing of $\mathscr{F}(s)$ only 20% longer than for pure diffusion

Brownian approach to translocation dynamics?

Kramers escape theory across free energy barrier $\mathscr{F}(s)$ delivers diffusive scaling:

$$\tau_T \simeq \frac{N^2}{D}$$

First remark: τ_T for crossing of $\mathscr{F}(s)$ only 20% longer than for pure diffusion

Second remark: Equil. time of free polymer estimated by diffusion over R_g:

$$\tau_R \simeq \frac{R_g^2}{D_{\mathrm{c.o.m.}}} \simeq \frac{R_g^2 N}{D_{\mathrm{mon}}} \simeq \frac{N^{1+2\nu}}{D_{\mathrm{mon}}}$$

Brownian approach to translocation dynamics?

Kramers escape theory across free energy barrier $\mathscr{F}(s)$ delivers diffusive scaling:

$$\tau_T \simeq \frac{N^2}{D}$$

First remark: τ_T for crossing of $\mathscr{F}(s)$ only 20% longer than for pure diffusion

Second remark: Equil. time of free polymer estimated by diffusion over R_g:

$$\tau_R \simeq \frac{R_g^2}{D_{\text{c.o.m.}}} \simeq \frac{R_g^2 N}{D_{\text{mon}}} \simeq \frac{N^{1+2\nu}}{D_{\text{mon}}}$$

Phantom chain ($\nu = 1/2$): $\tau_R \simeq N^2/D_{\text{mon}}$ marginal

Brownian approach to translocation dynamics?

Kramers escape theory across free energy barrier $\mathscr{F}(s)$ delivers diffusive scaling:

$$\tau_T \simeq \frac{N^2}{D}$$

First remark: τ_T for crossing of $\mathscr{F}(s)$ only 20% longer than for pure diffusion

Second remark: Equil. time of free polymer estimated by diffusion over R_g:

$$\tau_R \simeq \frac{R_g^2}{D_{\mathrm{c.o.m.}}} \simeq \frac{R_g^2 N}{D_{\mathrm{mon}}} \simeq \frac{N^{1+2\nu}}{D_{\mathrm{mon}}}$$

Phantom chain ($\nu = 1/2$): $\tau_R \simeq N^2/D_{\mathrm{mon}}$ marginal

SA chain ($\nu(2D) = 3/4$ and $\nu(3D) = 0.59$): τ_R longer than translocation time τ_T fl

J Chuang, Y Kantor & M Kardar, PRE (2001)

Non-Brownian translocation dynamics

Extended simulations in 1D: $\tau_T \simeq N^2$

In 2D: $\tau_T \simeq N^{2.5}$

Kantor & Kardar suggest $\tau_T \simeq N^{1+2\nu}$ like chain relax. time τ_R, but with large prefactor

Non-Brownian translocation dynamics

Extended simulations in 1D: $\tau_T \simeq N^2$

In 2D: $\tau_T \simeq N^{2.5}$

Kantor & Kardar suggest $\tau_T \simeq N^{1+2\nu}$ like chain relax. time τ_R, but with large prefactor

Scaling argument for anomalous dynamics:

1. Assume scaling $R_g \simeq N^{\nu}$ and chain relaxation time $\tau_R \simeq R_g^z$ /w dynamic expt. z

Non-Brownian translocation dynamics

Extended simulations in 1D: $\tau_T \simeq N^2$

In 2D: $\tau_T \simeq N^{2.5}$

Kantor & Kardar suggest $\tau_T \simeq N^{1+2\nu}$ like chain relax. time τ_R, but with large prefactor

Scaling argument for anomalous dynamics:

1. Assume scaling $R_g \simeq N^\nu$ and chain relaxation time $\tau_R \simeq R_g^z$ /w dynamic expt. z

2. Translocation progress has variance $\langle \Delta s^2(t) \rangle \simeq t^{2\zeta}$

Non-Brownian translocation dynamics

Extended simulations in 1D: $\tau_T \simeq N^2$

In 2D: $\tau_T \simeq N^{2.5}$

Kantor & Kardar suggest $\tau_T \simeq N^{1+2\nu}$ like chain relax. time τ_R, but with large prefactor

Scaling argument for anomalous dynamics:

1. Assume scaling $R_g \simeq N^\nu$ and chain relaxation time $\tau_R \simeq R_g^z$ /w dynamic expt. z

2. Translocation progress has variance $\langle \Delta s^2(t) \rangle \simeq t^{2\zeta}$

3. @ $\tau_T \simeq N^{\nu z}$ we should reach the value $N^2 \simeq \langle \Delta s^2(\tau_T) \rangle \simeq \tau_T^{2\zeta} \simeq N^{2\zeta\nu z}$

Non-Brownian translocation dynamics

Extended simulations in 1D: $\tau_T \simeq N^2$

In 2D: $\tau_T \simeq N^{2.5}$

Kantor & Kardar suggest $\tau_T \simeq N^{1+2\nu}$ like chain relax. time τ_R, but with large prefactor

Scaling argument for anomalous dynamics:

1. Assume scaling $R_g \simeq N^\nu$ and chain relaxation time $\tau_R \simeq R_g^z$ /w dynamic expt. z

2. Translocation progress has variance $\langle \Delta s^2(t) \rangle \simeq t^{2\zeta}$

3. @ $\tau_T \simeq N^{\nu z}$ we should reach the value $N^2 \simeq \langle \Delta s^2(\tau_T) \rangle \simeq \tau_T^{2\zeta} \simeq N^{2\zeta\nu z}$

4. $\rightsquigarrow \zeta = 1/(\nu z)$

Non-Brownian translocation dynamics

Extended simulations in 1D: $\tau_T \simeq N^2$

In 2D: $\tau_T \simeq N^{2.5}$

Kantor & Kardar suggest $\tau_T \simeq N^{1+2\nu}$ like chain relax. time τ_R, but with large prefactor

Scaling argument for anomalous dynamics:

1. Assume scaling $R_g \simeq N^\nu$ and chain relaxation time $\tau_R \simeq R_g^z$ /w dynamic expt. z
2. Translocation progress has variance $\langle \Delta s^2(t) \rangle \simeq t^{2\zeta}$
3. @ $\tau_T \simeq N^{\nu z}$ we should reach the value $N^2 \simeq \langle \Delta s^2(\tau_T) \rangle \simeq \tau_T^{2\zeta} \simeq N^{2\zeta\nu z}$
4. $\rightsquigarrow \zeta = 1/(\nu z)$
5. For diffusive translocation $z = (1+2\nu)/\nu$ and thus $\zeta = 1/(1+2\nu)$

Non-Brownian translocation dynamics

Extended simulations in 1D: $\tau_T \simeq N^2$

In 2D: $\tau_T \simeq N^{2.5}$

Kantor & Kardar suggest $\tau_T \simeq N^{1+2\nu}$ like chain relax. time τ_R, but with large prefactor

Scaling argument for anomalous dynamics:

1. Assume scaling $R_g \simeq N^\nu$ and chain relaxation time $\tau_R \simeq R_g^z$ /w dynamic expt. z

2. Translocation progress has variance $\langle \Delta s^2(t) \rangle \simeq t^{2\zeta}$

3. @ $\tau_T \simeq N^{\nu z}$ we should reach the value $N^2 \simeq \langle \Delta s^2(\tau_T) \rangle \simeq \tau_T^{2\zeta} \simeq N^{2\zeta\nu z}$

4. $\rightsquigarrow \zeta = 1/(\nu z)$

5. For diffusive translocation $z = (1+2\nu)/\nu$ and thus $\zeta = 1/(1+2\nu)$

6. $\rightsquigarrow \zeta(2D) = 0.4$ and $\zeta(3D) \approx 0.46$, i.e., subdiffusion

J Chuang, Y Kantor & M Kardar, PRE (2001)

Fractional approach to translocation dynamics

Pausing events of duration t distributed as

$$\psi(t) \simeq \frac{\tau^{\alpha}}{t^{1+\alpha}} \qquad \therefore \qquad \langle t \rangle = \int_0^{\infty} t\psi(t)dt \to \infty$$

Fractional approach to translocation dynamics

Pausing events of duration t distributed as

$$\psi(t) \simeq \frac{\tau^{\alpha}}{t^{1+\alpha}} \qquad \therefore \qquad \langle t \rangle = \int_0^{\infty} t\psi(t)dt \to \infty$$

Associated dynamic equation: Fractional Fokker-Planck equation

$$\frac{\partial}{\partial t}P(s,t) = {}_0D_t^{1-\alpha}\left(-v_\alpha \frac{\partial}{\partial s} + K_\alpha \frac{\partial^2}{\partial s^2}\right)P(s,t); \;\; {}_0D_t^{1-\alpha}f(t) = \frac{\partial}{\partial t}\frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(t')dt'}{(t-t')^{1-\alpha}}$$

Fractional approach to translocation dynamics

Pausing events of duration t distributed as

$$\psi(t) \simeq \frac{\tau^{\alpha}}{t^{1+\alpha}} \qquad \therefore \qquad \langle t \rangle = \int_0^{\infty} t\psi(t)dt \to \infty$$

Associated dynamic equation: Fractional Fokker-Planck equation

$$\frac{\partial}{\partial t}P(s,t) = {}_0D_t^{1-\alpha}\left(-v_\alpha\frac{\partial}{\partial s} + K_\alpha\frac{\partial^2}{\partial s^2}\right)P(s,t); \quad {}_0D_t^{1-\alpha}f(t) = \frac{\partial}{\partial t}\frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(t')dt'}{(t-t')^{1-\alpha}}$$

Moments:

$$\langle s(t) \rangle \simeq v_\alpha t^{\alpha} \overset{!}{\simeq} \sqrt{\langle \Delta s^2(t) \rangle} \qquad\qquad v_\alpha = 0: \quad \langle s^2(t) \rangle \simeq K_\alpha t^{\alpha}$$

Fractional approach to translocation dynamics

Pausing events of duration t distributed as

$$\psi(t) \simeq \frac{\tau^{\alpha}}{t^{1+\alpha}} \qquad \therefore \qquad \langle t \rangle = \int_0^{\infty} t\psi(t)dt \to \infty$$

Associated dynamic equation: Fractional Fokker-Planck equation

$$\frac{\partial}{\partial t}P(s,t) = {}_0D_t^{1-\alpha}\left(-v_\alpha\frac{\partial}{\partial s} + K_\alpha\frac{\partial^2}{\partial s^2}\right)P(s,t); \;\; {}_0D_t^{1-\alpha}f(t) = \frac{\partial}{\partial t}\frac{1}{\Gamma(\alpha)}\int_0^t \frac{f(t')dt'}{(t-t')^{1-\alpha}}$$

Moments:

$$\langle s(t)\rangle \simeq v_\alpha t^\alpha \overset{!}{\simeq} \sqrt{\langle \Delta s^2(t)\rangle} \qquad\qquad v_\alpha = 0: \quad \langle s^2(t)\rangle \simeq K_\alpha t^\alpha$$

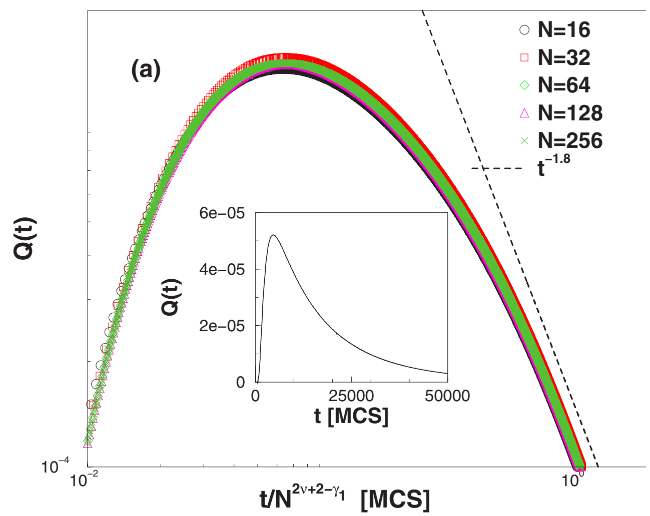
Mode relaxation:

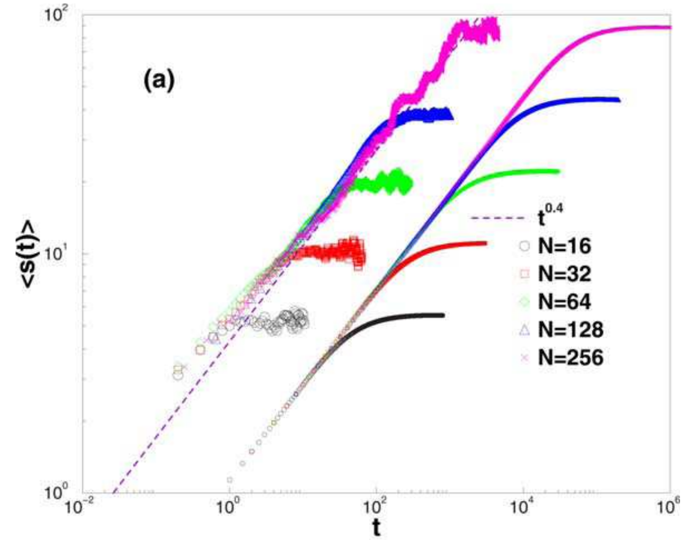
$$T_n(t) = t_n E_\alpha\Big(-\lambda_{n,\alpha}t^\alpha\Big) \sim t_n \begin{cases} 1 - t^\alpha/\Gamma(1+\alpha) \\ t^{-\alpha}/\Gamma(1-\alpha) \end{cases}$$

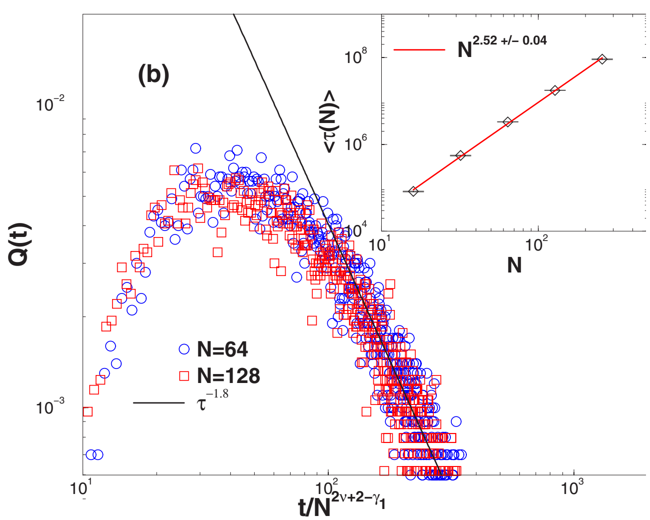
RM & J Klafter, Biophys J (2003); Review: RM & J Klafter, Phys Rep (2000) & J Phys A (2004)

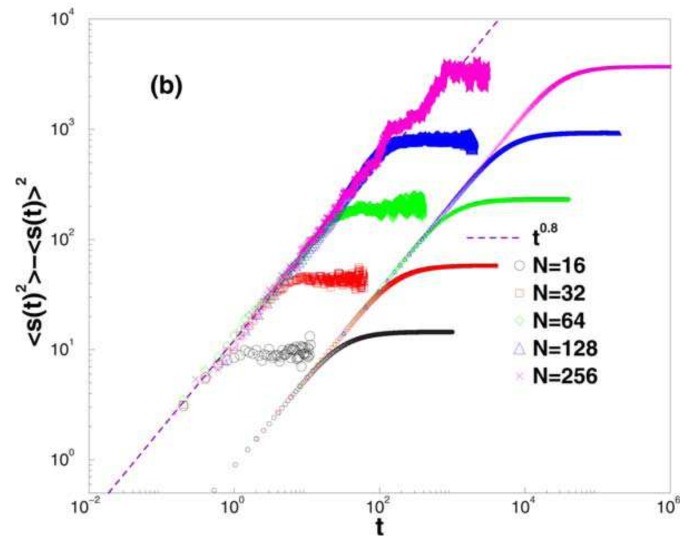
Agreement with simulations data: drift-free case

Theory prediction





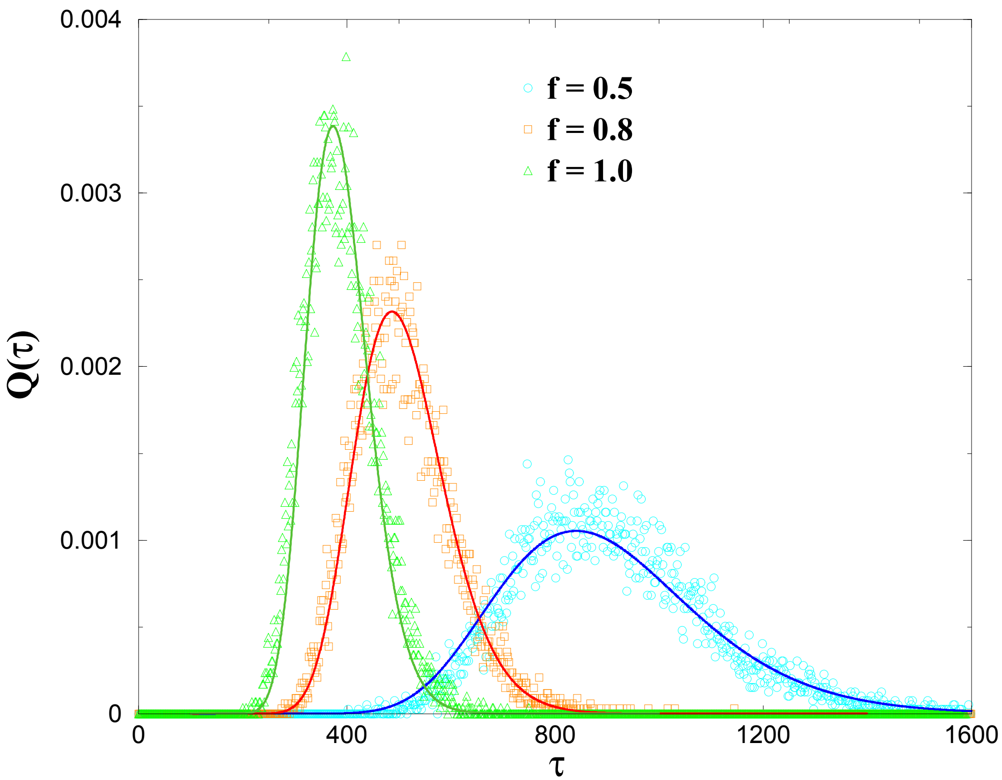


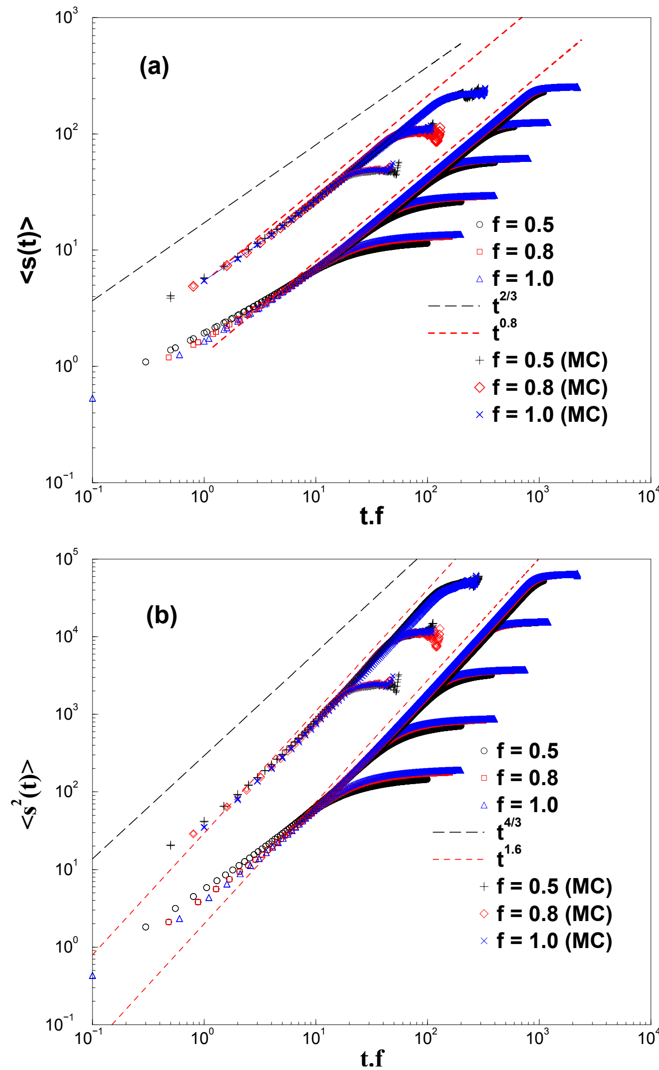


MC simulations: JLA Dubbeldam, A Milchev, VG Rostiashvili & TA Vilgis, PRE R.C. (2007)

Agreement with simulations data: constant drift case

First passage time distribution:





MC simulations: JLA Dubbeldam, A Milchev, VG Rostiashvili & TA Vilgis, PRE R.C. (2007)

Scaling laws for driven translocation

REM: Rouse relaxation time $\tau_R \simeq N^{1+2\nu}$

Scaling laws for driven translocation

REM: Rouse relaxation time $\tau_R \simeq N^{1+2\nu}$

¿How does the translocation time scale with N: $\tau_T \simeq N^{\alpha}$?

Scaling laws for driven translocation

REM: Rouse relaxation time $\tau_R \simeq N^{1+2\nu}$

¿How does the translocation time scale with N: $\tau_T \simeq N^{\alpha}$?

Kramers/Brownian approach: $\tau_T \simeq N^2$ unbiased, $\tau_T \simeq N$ with drift

Scaling laws for driven translocation

REM: Rouse relaxation time $\tau_R \simeq N^{1+2\nu}$

¿How does the translocation time scale with N: $\tau_T \simeq N^{\alpha}$?

Kramers/Brownian approach: $\tau_T \simeq N^2$ unbiased, $\tau_T \simeq N$ with drift

Estimate for lower bound under driving: $\tau_T \simeq N^{1+\nu}$ [Kantor & Kardar, PRE (2004)]

Scaling laws for driven translocation

REM: Rouse relaxation time $\tau_R \simeq N^{1+2\nu}$

¿How does the translocation time scale with N: $\tau_T \simeq N^\alpha$?

Kramers/Brownian approach: $\tau_T \simeq N^2$ unbiased, $\tau_T \simeq N$ with drift

Estimate for lower bound under driving: $\tau_T \simeq N^{1+\nu}$ [Kantor & Kardar, PRE (2004)]

Excellent agreement with simulations [Dubbeldam et al, EPL (2007); Gauthier & Slater, JCP (2007)]

Scaling laws for driven translocation

REM: Rouse relaxation time $\tau_R \simeq N^{1+2\nu}$

¿How does the translocation time scale with N: $\tau_T \simeq N^{\alpha}$?

Kramers/Brownian approach: $\tau_T \simeq N^2$ unbiased, $\tau_T \simeq N$ with drift

Estimate for lower bound under driving: $\tau_T \simeq N^{1+\nu}$ [Kantor & Kardar, PRE (2004)]

Excellent agreement with simulations [Dubbeldam et al, EPL (2007); Gauthier & Slater, JCP (2007)]

From theory with memory effects: $\tau_T \simeq N^{(1+2\nu)/(1+\nu)}$ [Vocks et al, J Phys Cond Mat (2008)]

Scaling laws for driven translocation

REM: Rouse relaxation time $\tau_R \simeq N^{1+2\nu}$

¿How does the translocation time scale with N: $\tau_T \simeq N^{\alpha}$?

Kramers/Brownian approach: $\tau_T \simeq N^2$ unbiased, $\tau_T \simeq N$ with drift

Estimate for lower bound under driving: $\tau_T \simeq N^{1+\nu}$ [Kantor & Kardar, PRE (2004)]

Excellent agreement with simulations [Dubbeldam et al, EPL (2007); Gauthier & Slater, JCP (2007)]

From theory with memory effects: $\tau_T \simeq N^{(1+2\nu)/(1+\nu)}$ [Vocks et al, J Phys Cond Mat (2008)]

Also corroborated by simulations [Vocks et al, J Phys Cond Mat (2008)]

Scaling laws for driven translocation

REM: Rouse relaxation time $\tau_R \simeq N^{1+2\nu}$

¿How does the translocation time scale with N: $\tau_T \simeq N^\alpha$?

Kramers/Brownian approach: $\tau_T \simeq N^2$ unbiased, $\tau_T \simeq N$ with drift

Estimate for lower bound under driving: $\tau_T \simeq N^{1+\nu}$ [Kantor & Kardar, PRE (2004)]

Excellent agreement with simulations [Dubbeldam et al, EPL (2007); Gauthier & Slater, JCP (2007)]

From theory with memory effects: $\tau_T \simeq N^{(1+2\nu)/(1+\nu)}$ [Vocks et al, J Phys Cond Mat (2008)]

Also corroborated by simulations [Vocks et al, J Phys Cond Mat (2008)]

¿ Can we reconcile these conflicting statements ?

Scaling laws for driven translocation

REM: Rouse relaxation time $\tau_R \simeq N^{1+2\nu}$

¿How does the translocation time scale with N: $\tau_T \simeq N^{\alpha}$?

Kramers/Brownian approach: $\tau_T \simeq N^2$ unbiased, $\tau_T \simeq N$ with drift

Estimate for lower bound under driving: $\tau_T \simeq N^{1+\nu}$ [Kantor & Kardar, PRE (2004)]

Excellent agreement with simulations [Dubbeldam et al, EPL (2007); Gauthier & Slater, JCP (2007)]

From theory with memory effects: $\tau_T \simeq N^{(1+2\nu)/(1+\nu)}$ [Vocks et al, J Phys Cond Mat (2008)]

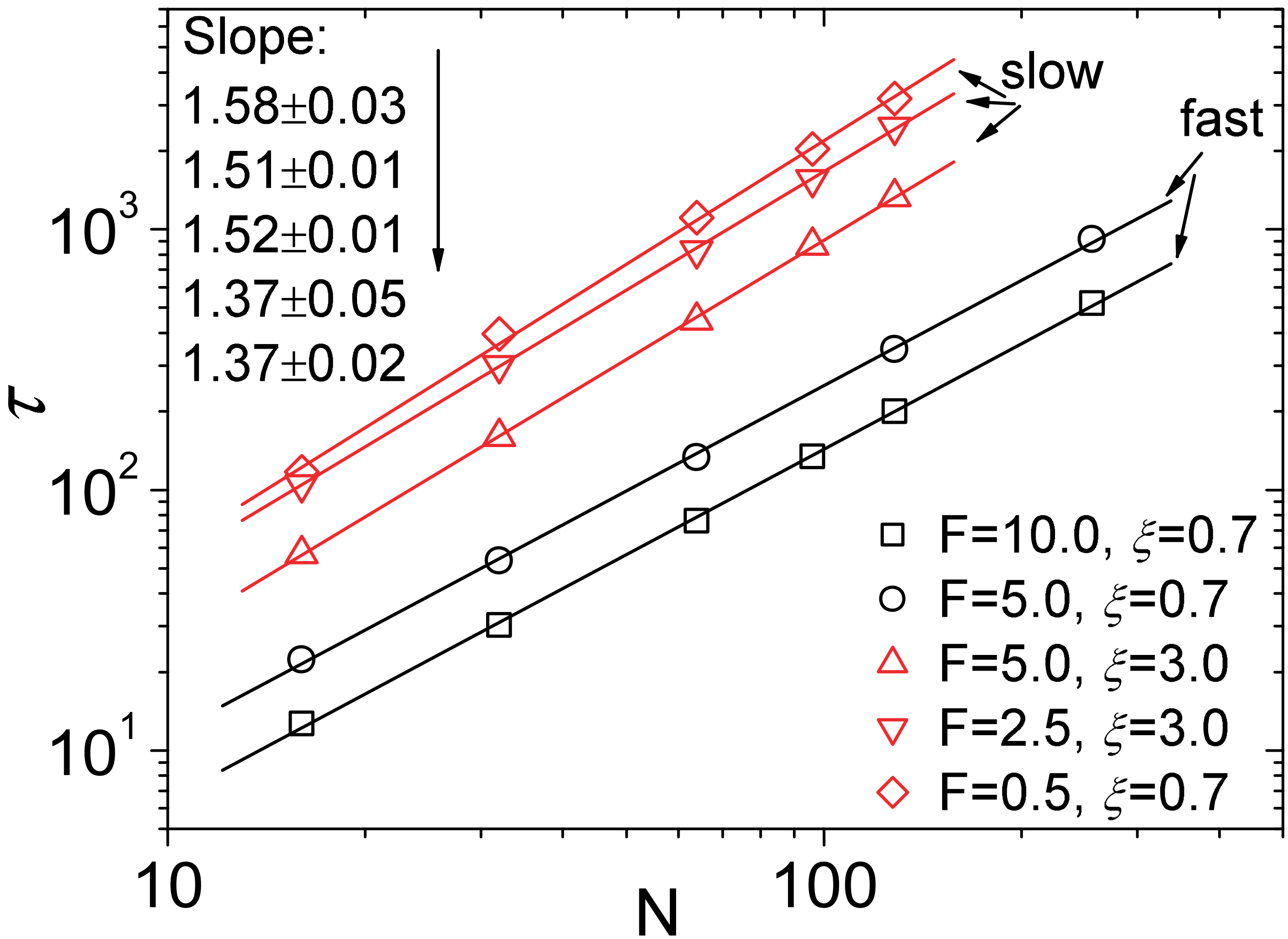
Also corroborated by simulations [Vocks et al, J Phys Cond Mat (2008)]

¿ Can we reconcile these conflicting statements ?

Yes, we can . . .

K Luo, T Ala-Nissilä, S-C Ying & RM, EPL (2009)

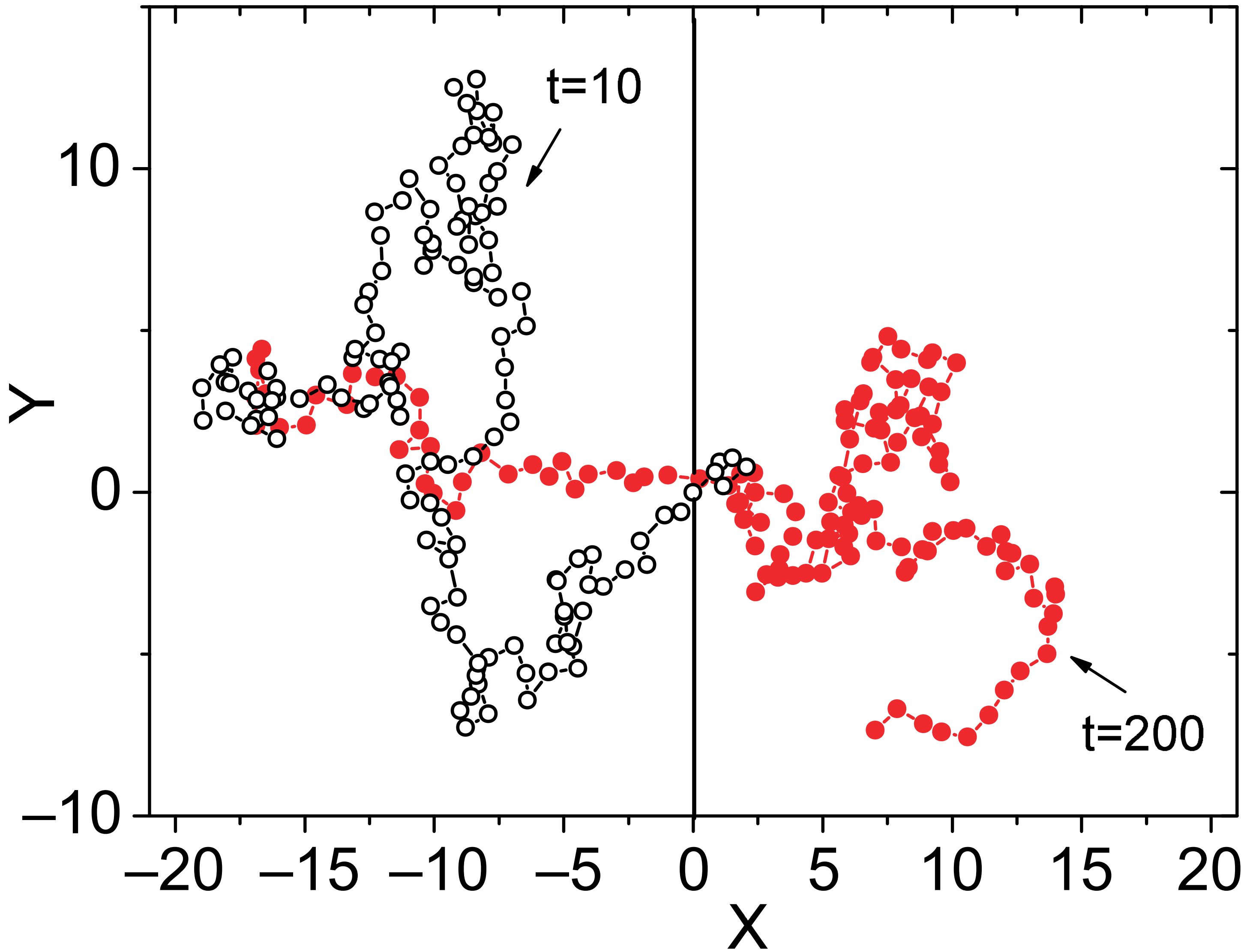
Slow versus fast translocation



In 3D interpolation between $\alpha = 1 + \nu \approx 1.59$ and $\alpha' = (1 + 2\nu)/(1 + \nu) \approx 1.37$

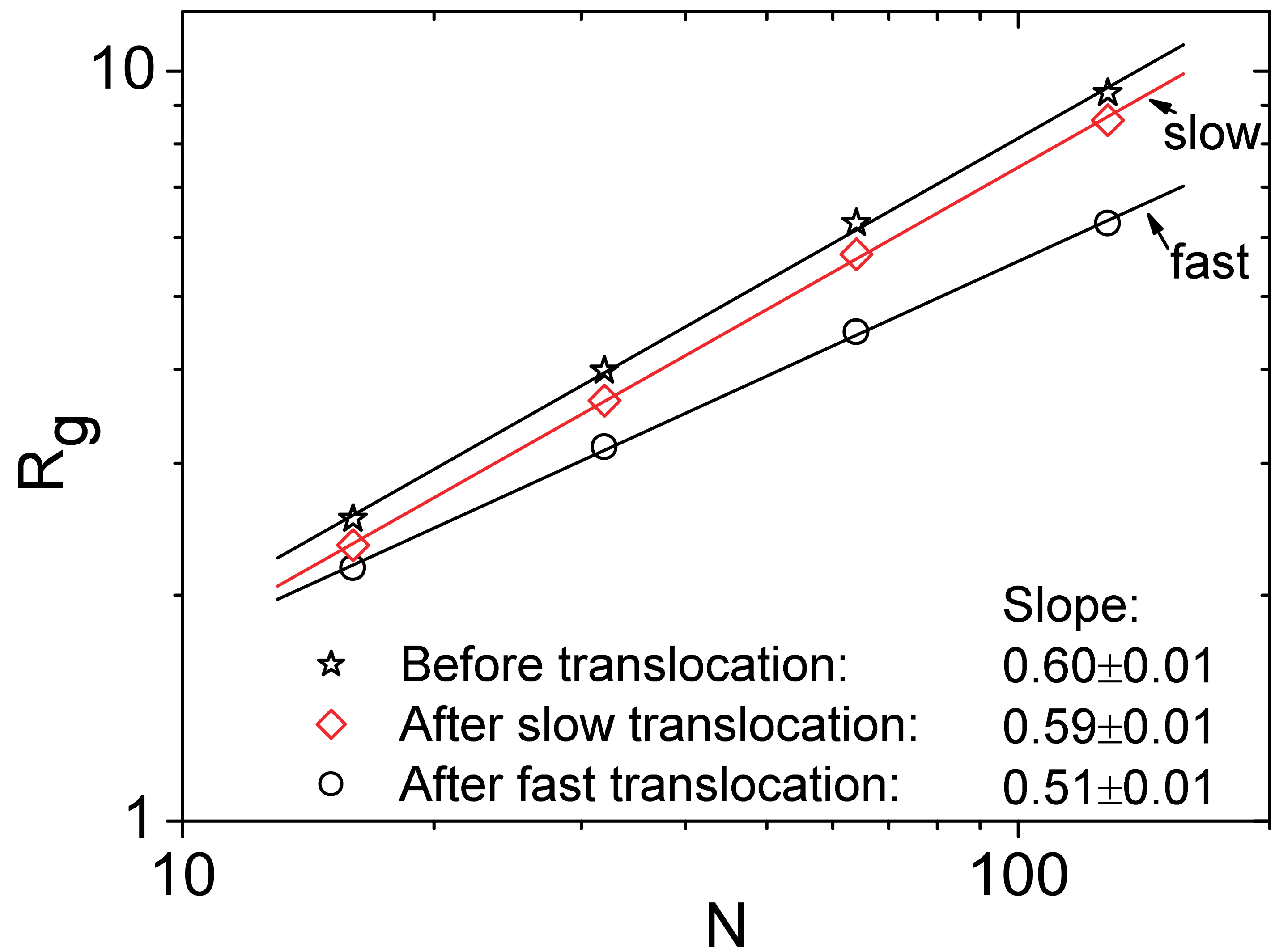
K Luo, T Ala-Nissilä, S-C Ying & RM, EPL (2009)

Fast translocation: nonequilibrium effects



K Luo, T Ala-Nissilä, S-C Ying & RM, EPL (2009)

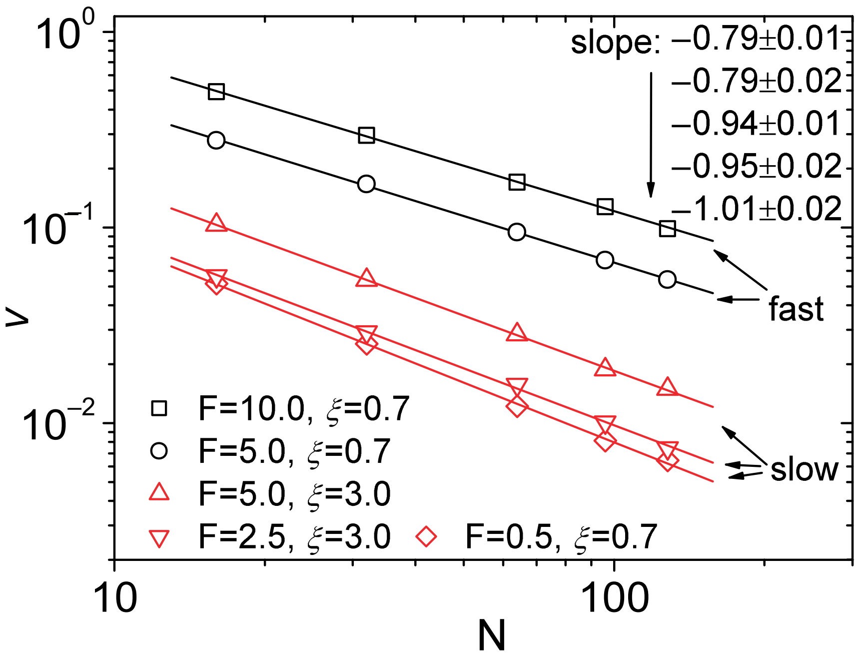
Fast translocation: nonequilibrium effects

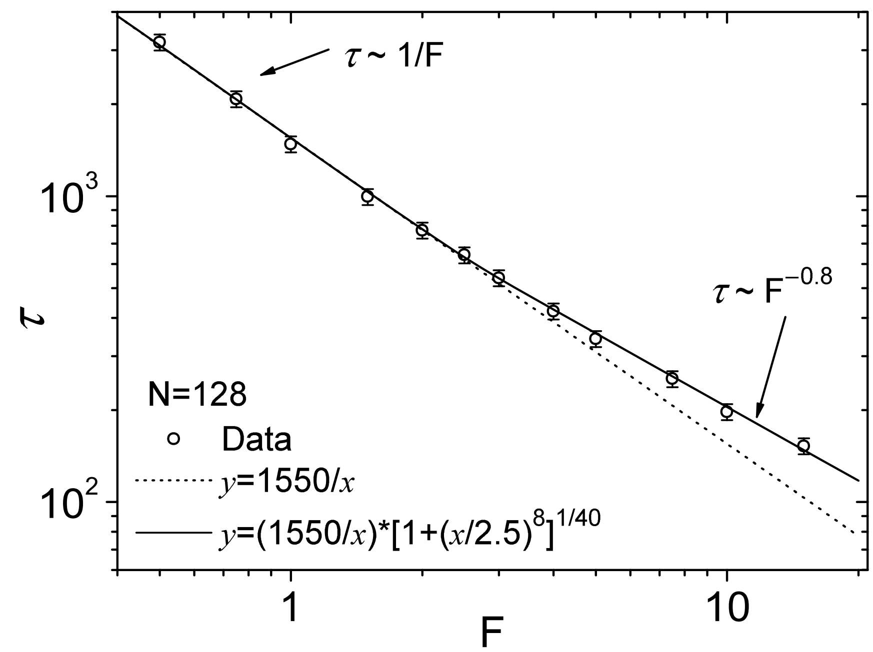


K Luo, T Ala-Nissilä, S-C Ying & RM, EPL (2009)

Fast translocation: nonequilibrium effects

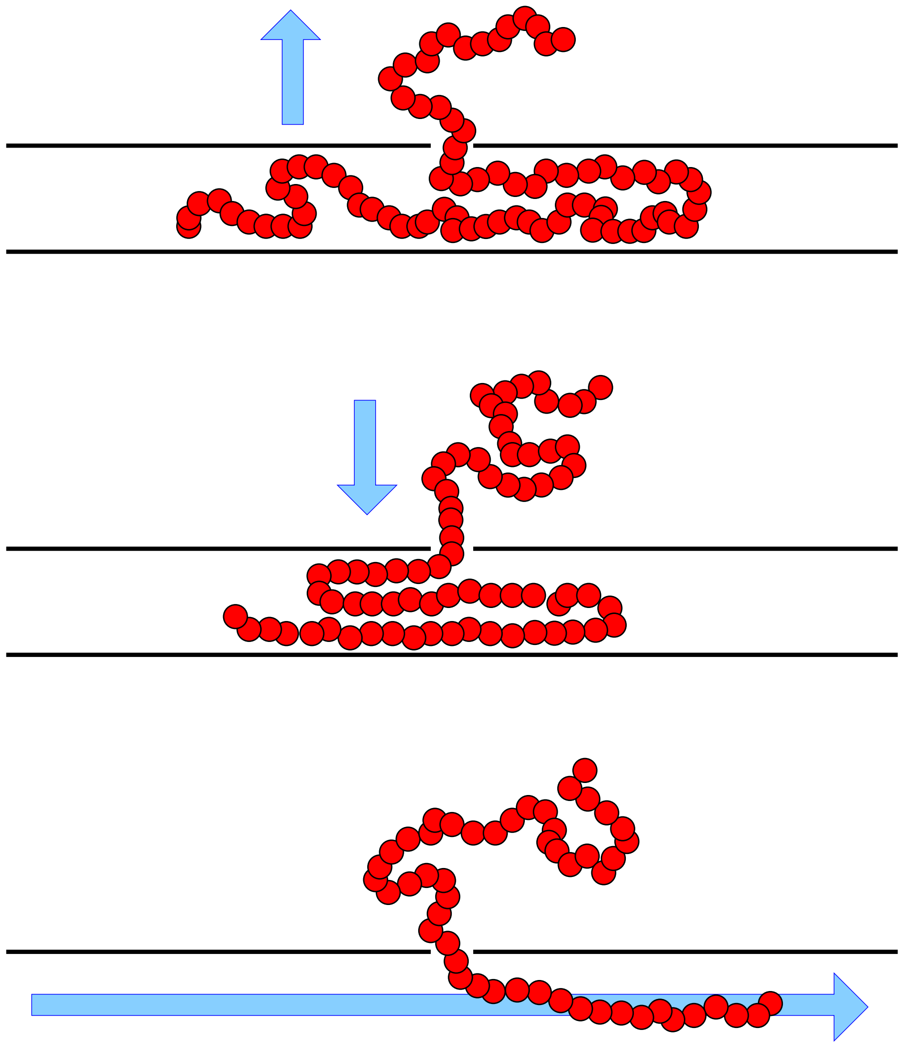
	F	ξ	F/ξ	α ($\tau \sim N^{\alpha}$)	δ ($v \sim N^{\delta}$)	β ($\langle s(t)\rangle \sim t^{\beta}$)	$\alpha\beta$
Fast	10.0	0.7	14.28	1.37 ± 0.02	-0.79 ± 0.01	0.84 ± 0.01	1.15
	5.0	0.7	7.14	1.37 ± 0.05	-0.79 ± 0.02	0.85 ± 0.01	1.16
Slow	5.0	3.0	1.67	1.52 ± 0.01	-0.94 ± 0.01	0.71 ± 0.01	1.08
	2.5	3.0	0.83	1.51 ± 0.02	-0.95 ± 0.02	0.69 ± 0.01	1.04
	0.5	0.7	0.71	1.58 ± 0.03	-1.01 ± 0.02	0.64 ± 0.01	1.01





K Luo, T Ala-Nissilä, S-C Ying & RM, EPL (2009)

Variations on a theme



Out of confinement

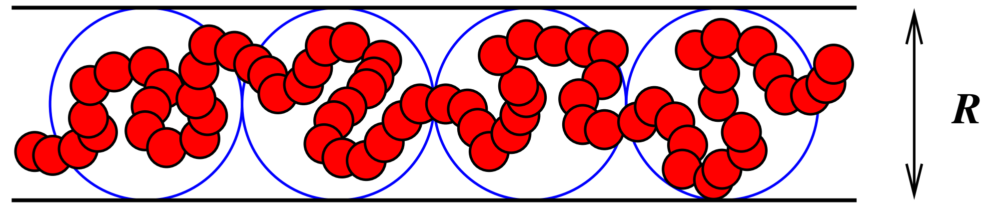
Into confinement

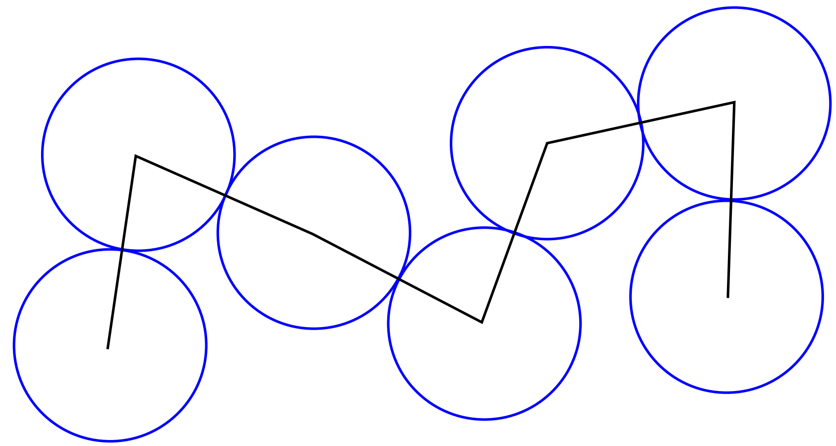
The chain sucker

K Luo, RM, T Ala-Nissilä & S-C Ying, PRE (2009); K Luo & RM, JCP (2010), PRE (2010)

Translocation into a laterally unbounded confinement

Chain /w N monomers of diam. σ

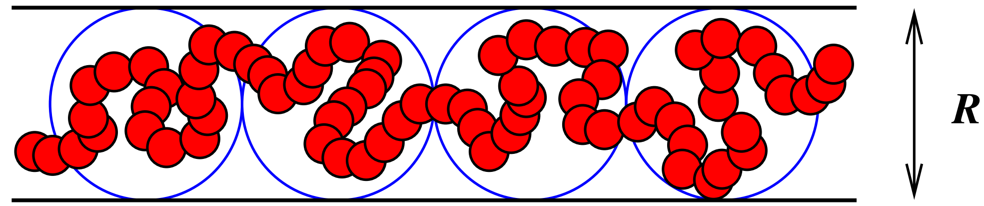


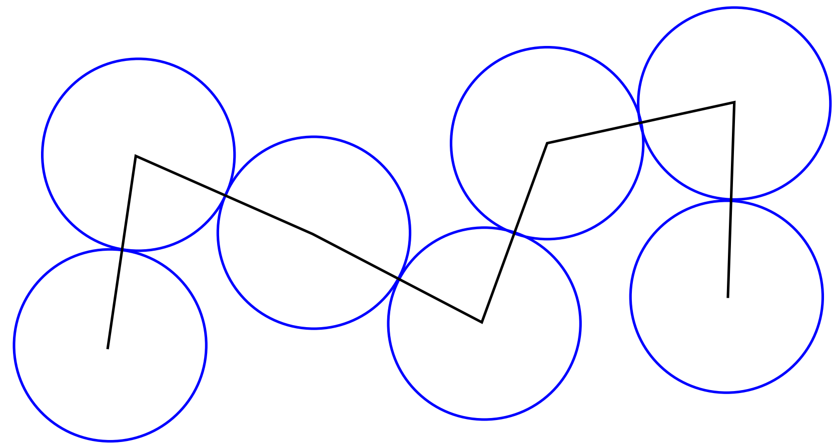


Translocation into a laterally unbounded confinement

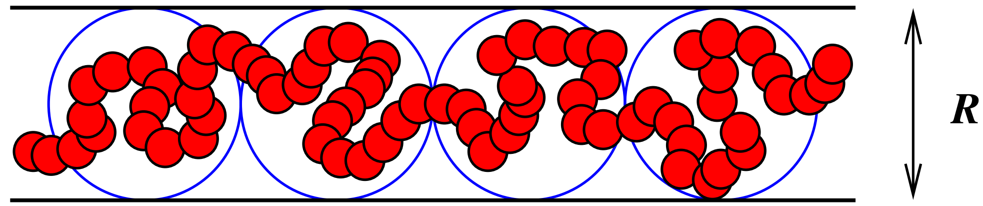
Chain /w N monomers of diam. σ

$\sigma \ll R \ll R_g$: 2D SAW of n_b blobs





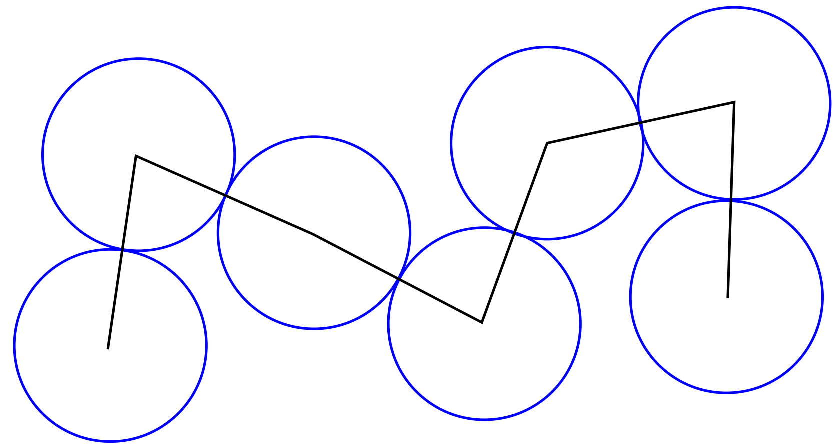
Translocation into a laterally unbounded confinement



Chain /w N monomers of diam. σ

$\sigma \ll R \ll R_g$: 2D SAW of n_b blobs

$$g = \left\{ \begin{array}{c} \#\text{monom.} \\ \text{per blob} \end{array} \right\} = \left(\frac{R}{\sigma}\right)^{1/\nu_{3D}}$$



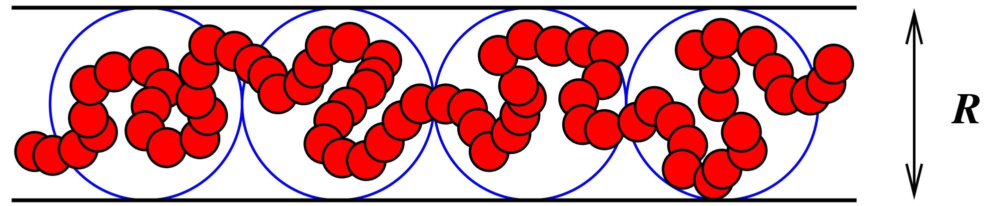
Translocation into a laterally unbounded confinement

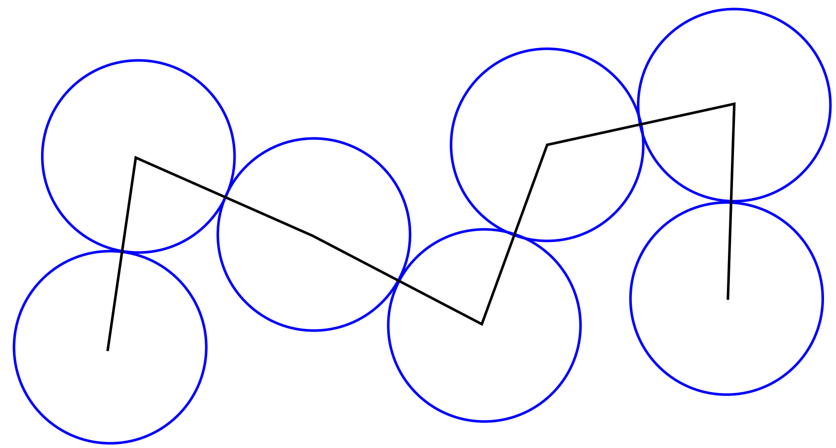
Chain /w N monomers of diam. σ

$\sigma \ll R \ll R_g$: 2D SAW of n_b blobs

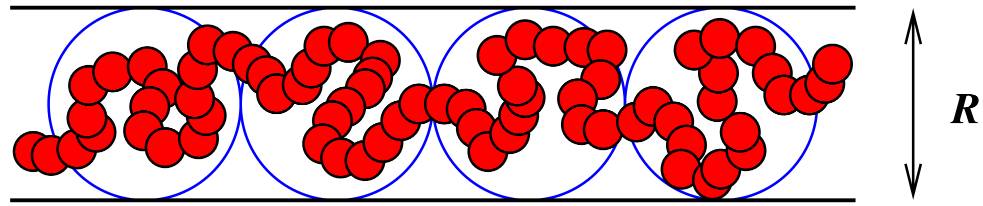
$$g = \left\{ \begin{array}{c} \text{\#monom.} \\ \text{per blob} \end{array} \right\} = \left(\frac{R}{\sigma} \right)^{1/\nu_{3D}}$$

$$n_b = \left\{ \text{\#blobs} \right\} = \frac{N}{g}$$



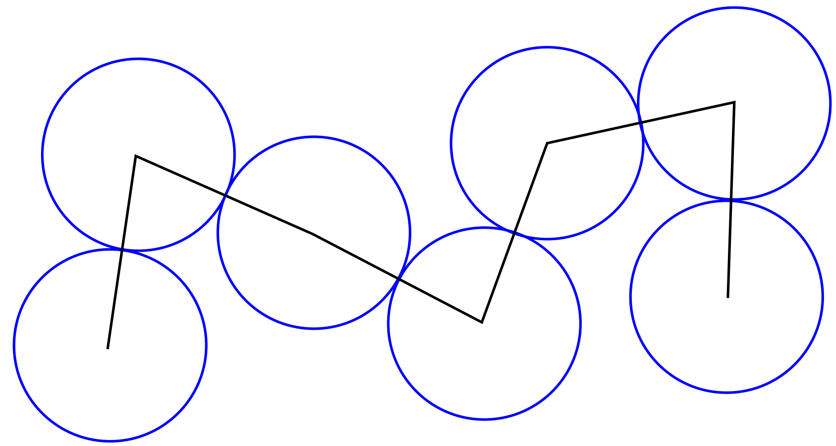


Translocation into a laterally unbounded confinement



Chain /w N monomers of diam. σ

$\sigma \ll R \ll R_g$: 2D SAW of n_b blobs



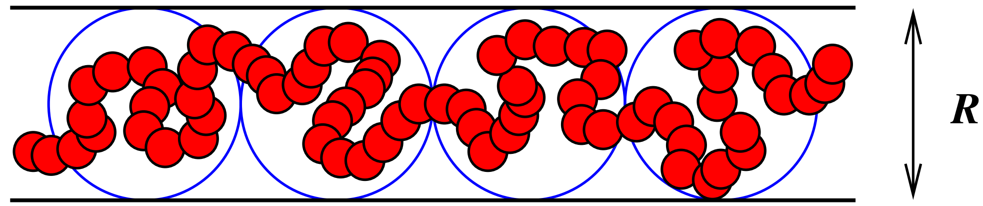
$$g = \left\{ \begin{array}{c} \#\text{monom.} \\ \text{per blob} \end{array} \right\} = \left(\frac{R}{\sigma}\right)^{1/\nu_{3D}}$$

$$n_b = \left\{ \#\text{blobs} \right\} = \frac{N}{g}$$

Longitudinal size of polymer:

$$R_{\parallel} \simeq n_b^{\nu_{2D}} R \simeq N^{\nu_{2D}} \sigma \left(\frac{\sigma}{R}\right)^{\nu_{2D}/\nu_{3D}-1} \simeq N^{3/4} \sigma \left(\frac{\sigma}{R}\right)^{0.28}$$

Translocation into a laterally unbounded confinement

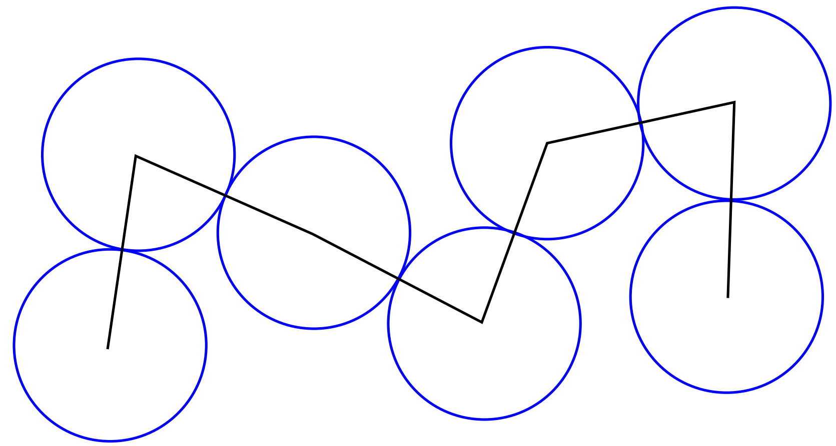


Chain /w N monomers of diam. σ

$\sigma \ll R \ll R_g$: 2D SAW of n_b blobs

$$g = \left\{ \begin{array}{c} \#\text{monom.} \\ \text{per blob} \end{array} \right\} = \left(\frac{R}{\sigma}\right)^{1/\nu_{3D}}$$

$$n_b = \left\{\#\text{blobs}\right\} = \frac{N}{g}$$



Longitudinal size of polymer:

$$R_\parallel \simeq n_b^{\nu_{2D}} R \simeq N^{\nu_{2D}} \sigma \left(\frac{\sigma}{R}\right)^{\nu_{2D}/\nu_{3D}-1} \simeq N^{3/4} \sigma \left(\frac{\sigma}{R}\right)^{0.28}$$

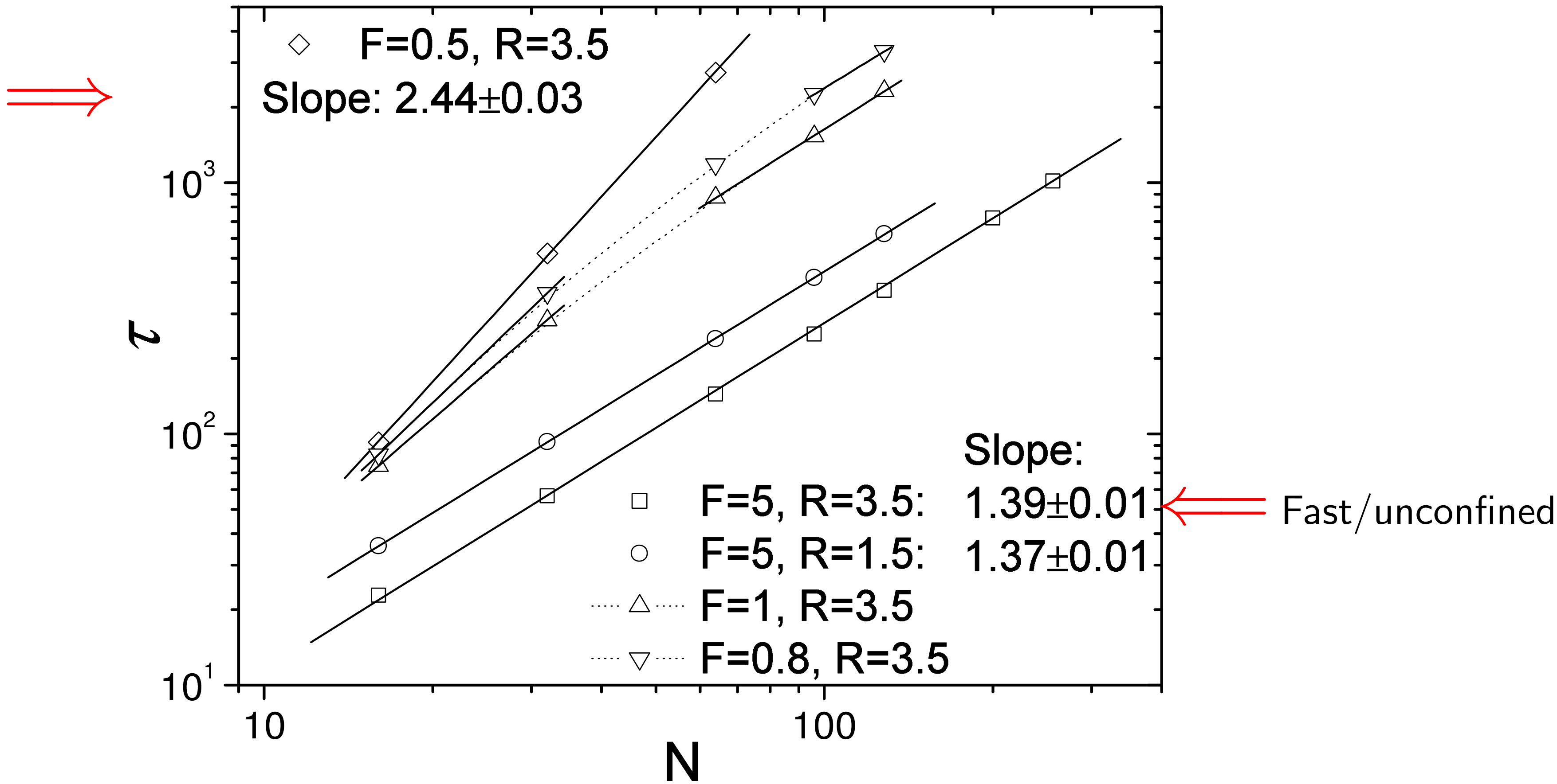
Longit. relax. time: polymer moves by its own size (diffusivity $D \simeq 1/N$):

$$\tau_\parallel \simeq \frac{R_\parallel^2}{D} \simeq N^{1+2\nu_{2D}} R^{2(1-\nu_{2D}/\nu 3D)} \simeq N^{2.50} R^{-0.55}$$

K Luo, T Ala-Nissilä, S-C Ying & RM, PRE (2009)

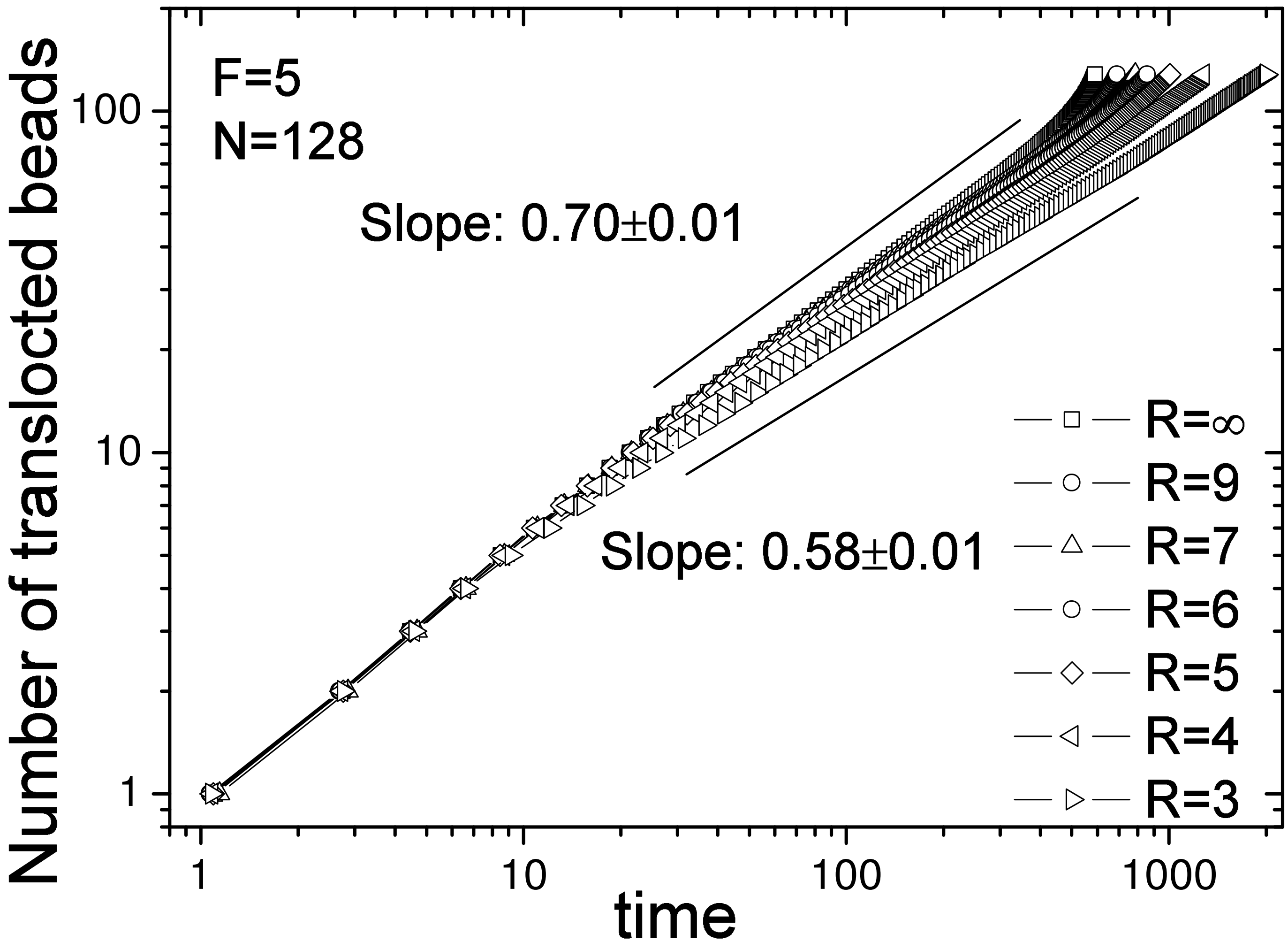
Translocation into a laterally unbounded confinement

$$\tau_{\parallel} \simeq \frac{R_{\parallel}^2}{D} \simeq N^{1+2\nu_{2D}} R^{2(1-\nu_{2D}/\nu 3D)} \simeq N^{2.50} R^{-0.55}$$



K Luo, T Ala-Nissilä, S-C Ying & RM, PRE (2009)

Translocation into a laterally unbounded confinement



K Luo, T Ala-Nissilä, S-C Ying & RM, PRE (2009)