For the assessment of bone stage (e.g., regarding different osteoporotic stages), usually the bone mineral density (BMD) is measured. However, this measurement does not contain any information about the structures inside the bone. Recent work emphasized the importance of analyzing the structural changes of trabecular bone. Different approaches for the study of trabecular bone were successfully introduced for 2D image analysis, as measures of complexity based on symbolic dynamics.

The new available 3D bone images (e.g., CT-data) challenge the development of new 3D measures of complexity, which are able to assess structural changes in trabecular bone. We consider bone new developments of 3D measures that are based on lacunarity, Moran’s index and geometrical properties.

Lacunarity
Dougherty & Honebry (2001) suggested the lacunarity measure for the structural assessment of 2D bone images. This measure can be simply applied to 3D images and is defined as:

\[ L(r) = \frac{\mu_2}{\mu_1} \]

where

\[ \mu_1 = \frac{1}{N} \sum n(s,r), \]
\[ \mu_2 = \frac{1}{N} \sum n(s,r)^2, \]

are the first and second moments of the mass distributions \( n(s,r) \) determined in equally spaced boxes of size \( r \) over the 3D image. Using the lacunarity and the complementary lacunarity (i.e., the lacunarity computed from the complementary objects, in our case marrow), we define the normalized lacunarity by

\[ \Lambda'(r) = 2 - \left( \frac{1}{\Lambda(r)} + \frac{1}{\Lambda(r)} \right) \]

ensuring measures between zero and one (from less lacunarity to high lacunarity, or from high translational invariance to less translational invariance).

Shape Index
Here we introduce a new index, which measures the surface of the trabecular bone within small cubic boxes and normalizes this value by the minimal surface possible for this bone volume

\[ \frac{S}{\sqrt{36\pi V^2}} \]

where \( S \) is bone surface and \( V \) is bone volume. The mean value and the entropy of this measure represent the shape of the bone structures – values of \( S \) smaller than one reveal concave structures (the smaller the value the higher the amount of concave structures).

Preliminary Results
The lacunarity shows only a small dependence on different osteoporotic stages (corresponds with a decrease of BTV/Ty), revealing small changes in the translational invariance during bone loss. However, this measure plotted as a function of the length scale provides some interesting features, which will be further investigated.

The Moran’s index reveals an increase of autocorrelation for decreasing bone. This is a hint for a loss of plate-like structures in the trabecular bone.

Moran’s Index
The Moran’s index \( I \) was introduced for measuring the two-dimensional spatial autocorrelation of a population in an eco-system, but was successfully applied to 2D image analysis (Chen et al., 2003).

We extend this common measure to 3D as

\[ I = \frac{N}{\sum_{i=1}^{N} \sum_{j=1}^{N} d_i d_j (x_i - \bar{x})(x_j - \bar{x})} \]

where \( d_i \) is the geometric size of the analyzed object, \( N \) is the total number of voxels and \( \delta \) is the number of contiguous pairs. This measure ranges between –1 and 1 (from high autocorrelation to high anti-autocorrelation).

References


N. Marwan, eMail: marwan@agnld.uni-potsdam.de
T el.: 0049 (0) 331 977-1302
University of Potsdam, Germany
Nonlinear Dynamics Group
Norbert Marwan