Recurrence Quantification Analysis

For the investigation of dynamical systems, recurrence based methods have proven its potential even for short and non-stationary data series. A recurrence plot is usually defined as a binary matrix representing the pairwise distances between the values of a data series. \( R(x) = D(x - \tau x) \times D(x - \tau y) \).

By quantification of the line structures in a recurrence plot, we are able to characterise the dynamics of the system with much complexity. It has been shown that such measures, calculated in moving windows, are able to detect transitions in the dynamics of systems, like chaos-period, chaos-chaos and chaos-SNA transitions (Marwan et al. 2007).

Here we exemplary use the measures of complexity measuring the fraction of recurrence points forming diagonal (Determinism, DET) and vertical lines (Laminarity, LAM). For both measures, we need the histograms of line lengths \( P(l) \). Determinism is the probability that recurrence states further be recurrent and Laminarity is the probability that very slowly changing states remain in similar states. High values of Determinism are typical for deterministic systems and high values of Laminarity are typical for intermittency.

Application to Event Related Potentials (ERP)

In the Oddball experiment, a number of visual or acoustic stimuli of different surprising effect (10% and 90% event probability) is shown to a proband. The averaging of the measured EEG data reveals a P300 component, which is anti-correlated with the event probability. This component reflects the switching between two modi of cognitive behaviour: During episodes where the frequent stimuli are presented to the subjects, they went into a mode of automatic processing of the events. When suddenly the rare stimulus occurs, the brain function is switched to controlled processing.

The investigation of such ERPs on a single trial basis is rather difficult. However, recurrence-based methods have the potential to recognize the specific ERP components even on a single trial basis (Marwan and Maikne, 2004; Marwan et al. 2007; Schinkel et al. 2007).

Bootstrap Procedure

As a statistical test for the RQA based transition analysis we propose the following bootstrap procedure:

1. Merge all local histograms of line lengths \( P(l) \).
2. Now we draw \( n \) line structures from \( Q(l) \), \( n \) is the mean number of line lengths \( l \).
3. We use \( P(l) \) for calculating our RQA measure (DET and LAM, resp.).
4. Repeating steps (2) and (3) we get empirical test distributions for DET and LAM, which we use for statistical tests (e.g. quantiles). In the following examples we use 5,000 realisations.

Application: Event Related Potentials

Applying RQA on EEG measurements of an Oddball experiment, we find event related potentials (P300) even in single trials (Marwan and Maikne, 2004; Schinkel et al. 2007). Recurrence plots of EEG signals measured (A) without surprise and (B) with surprise. RQA settings: no embedding, fixed RR (5%), window = 200 and orange bars). 99% confidence bounds are shown as blue dash-dotted lines. RQA settings: no embedding, fixed RR (5%), window = 200 and step = 200.

Empirical distributions for DET and LAM derived from bootstrapping recurrence structures. These distributions follow normal distributions (a fitted normal distribution shown by the black line).

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