The influence of particle adhesion on the stability of agglomerates in Saturn’s rings

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Abstract

In planetary rings, binary collisions and mutual gravity are the predominant particle interactions. Based on a viscoelastic contact model we implement the concept of static adhesion. We discuss the collision dynamics and obtain a threshold velocity for restitution or agglomeration to occur. The latter takes place within a range of a few cm s\(^{-1}\) for icy grains at low temperatures. The stability of such two-body agglomerates bound by adhesion and gravity in a tidal environment is discussed and applied to the saturnian system. A maximal agglomerate size for a given orbit location is obtained. In this way we are able to resolve the borderline of the zone where agglomerates can exist as a function of the agglomerate size and thus gain an alternative to the classical Roche limit. An increasing ring grain size with distance to Saturn as observed by the VIMS-experiment on board the Cassini spacecraft can be found by our estimates and implications for the saturnian system will be addressed.

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1. Introduction

Planetary rings are composed of myriads of grains revolving around their central planets in almost Keplerian orbits. Measurements on one hand and analytical studies on the other give insights about the material, the size, and the velocity distribution of the ring particles, which, however, are still poorly known.

Ring systems can be considered as granular gases under Keplerian shear where the granules mainly interact via binary, dissipative collisions and gravity. A quasi-equilibrium granular temperature, determined by granular cooling and viscous heating, ensures the stability of a planetary ring and thus crucially determines its dynamical lifetime and shape (Goldreich and Tremaine, 1978; Schmidt et al., 1999). Experimental and theoretical studies have been undertaken to account for the role of dissipation during collision in a qualitative and quantitative way. The amount of dissipated energy is quantified by the coefficient of restitution, which denotes the ratio of the relative velocity before and after collision. For perfectly smooth solid ice spheres the restitution coefficient has been obtained numerically and analytically (Dilley, 1993; Brilliantov et al., 1996; Thornton, 1997) and measured in lab experiments (Bridges et al., 1984; Hatzes et al., 1988; Supulver et al., 1995; Dilley and Crawford, 1996; Higa et al., 1998) with respect to an application to planetary rings. Experiments that studied collisions of frost covered icy particles (Hatzes et al., 1991; Bridges et al., 1996; Supulver et al., 1997) observed not only the restitution but also an agglomeration of grains under certain impact conditions. Theoretical considerations can account for both physical scenarios, restitution and agglomeration, if attractive forces are considered (e.g., Johnson et al., 1971; Thornton, 1997; Hertzsch, 2002). The coexistence of agglomeration and restitution crucially determines the evolution of a particle ensemble (Spahn et al., 2004). Growth processes in planetary rings and the existence of satellites are limited by tidal stresses. The Roche limit \(r_{\text{Roche}}\) is the closest distance from a planet of radius \(R_{\text{planet}}\) and density \(\rho_{\text{planet}}\) at which a homogeneous, self-gravitating liquid satellite of density \(\rho_{\text{satellite}}\) can exist in a circular orbit and in synchr-
nous rotation, where,

$$\frac{R_{\text{planet}}}{R_{\text{satellite}}} = 2.456 \left( \frac{\rho_{\text{planet}}}{\rho_{\text{satellite}}} \right)^{1/3} \quad (1)$$

(Roche, 1847; Chandrasekhar, 1969). Since the saturnian ring system is thought to consist of icy particles ($\rho = 10^3$ kg m$^{-3}$), the corresponding classical Roche limit is located at 2.2 Saturn radii (mid A-ring). Weidenschilling et al. (1984) considered a vanishing net attraction between two solid ring particles and the effects of different agglomerate proportion, shape, and rotation in orbit about a planet. But as long as gravitation is considered alone these criteria do only depend on the distance from the planet and the size ratio of colliding particles (Weidenschilling et al., 1984; Ohhtski, 1993). Particle assemblies held together by sticking forces as observed in lab experiments are possible (Tremaine, 2003), but have so far not been considered in N-body simulations. While the disruption of a solid body with finite internal strength has been analyzed in various studies and a critical distance to a central body as a function of the satellite’s size has been obtained (Aggarwal and Oberbeck, 1974; Dobrovolski, 1990; Davidson, 1999, 2001) there is furthermore evidence from the visual and infrared mapping spectrometer (VIMS) of the Cassini spacecraft (http://photojournal.jpl.nasa.gov/catalog/PIA06349), that grain sizes increase with increasing distance to Saturn.

Here we include adhesion in the viscoelastic binary collision dynamics. The resulting agglomeration and restitution will be discussed. An orbital limit distance for adhesive agglomerates to be stable will be derived as a function of the aggregate size and compared with the commonly known Roche limit. Influences on the size distribution of planetary rings and thus otherwise purely gravitationally dominated systems are discussed, where we will focus on the outer saturnian system.

2. Collision dynamics

Apart from any collisions gravitationally bound grains revolve around a central body in elliptic orbits. Their binary interactions sensitively influence this dynamics. Particle interactions contain long-range forces, such as mutual gravity or electrostatic forces, and short-range interactions such as contact forces acting during a physical contact of two particles. In case of Saturn’s rings, particles mainly interact via collisions and their mutual gravity. Any binary collision can either result in (i) agglomeration, (ii) restitution, or (iii) fragmentation.

Agglomeration becomes possible if attractive forces and dissipation dominate repulsive ones allowing for aggregation, while fragmentation occurs if relative velocities are large enough to erode sticking bonds between particles themselves. In all three cases (i)–(iii), mechanical energy is dissipated, where only the relative motion of the two grains is affected. Thus, we define the relative coordinate to be

$$\tilde{d}(t) = \tilde{r}_2(t) - \tilde{r}_1(t) = [(R_1 + R_2) - \xi(t)] \tilde{e}_d, \quad (2)$$

where $\xi(t)$ denotes the current penetration depth, if the particles are in contact ($\xi > 0$) and the distance between the particle surfaces otherwise ($\xi \leq 0$).

Since ring particles presumably consist of ice–silicate composites covered by regolith layers, impact velocities are small compared to the corresponding speed of sound. No solid body waves will be excited if the impact speeds of solid particles are low and an impact can thus be considered as adiabatic. A possible fragmentation for high impact speeds as given in Higa et al. (1998) will not be considered in the following discussion and denotes an upper applicability limit for the presented model. Based on the contact dynamics of two solid spheres of arbitrary size (Hertz, 1882), viscoelastic particle properties have been incorporated as an origin of dissipative effects (Hertzsch et al., 1995; Brilliantov et al., 1996). This viscoelastic approach, which explains lab experiments with icy spheres (Bridges et al., 1996) fairly well, denotes the base of the presented collision dynamics. Effects of adhesion on static elastic particle contacts have been discussed in Johnson (1971). By introducing a surface energy $\gamma S = -\pi \gamma a^2$, with $\gamma$ being the surface energy per unit area of both spheres and $a$ the contact radius, an adhesive contact increases the apparent load compared to a pure elastic contact. A resulting adhesive bond

$$\tilde{F}_{\text{ad}} = \frac{3}{2} \pi \gamma R_{\text{eff}} \tilde{e}_d. \quad (3)$$

where $R_{\text{eff}} = R_1 R_2 / (R_1 + R_2)$ denotes the effective particle radius is independent of the elastic material properties. Specific surface structures can only be accounted for in terms of $\gamma$. Larger contact areas result in a higher surface energy and thus the adhesive bond strength is linearly dependent on the effective grain radius, favoring smaller grains sitting on larger ones. Then, the particle deformation $\xi$ is related to the contact radius $a$ (Attard and Parker, 1992) by

$$\xi = \frac{a^2}{R_{\text{eff}}} - \sqrt{\frac{2 \pi \gamma}{E_{\text{eff}}}} a^{1/2}. \quad (4)$$

In case of a pure elastic contact Eq. (4) reduces to the Hertzian relation $a^2 = \xi R_{\text{eff}}$. The Hertzian elastic constant $H = 4 E_{\text{eff}} / R_{\text{eff}}^{3/2}$ accounts for material properties, where the Young modulus $Y$ and the Poisson ratio $\nu$ give the combined constant $E = Y / (1 - \nu^2)$ and its effective value $E_{\text{eff}} = E_1 E_2 / (E_1 + E_2)$. For solid particles larger than mm the second term of Eq. (4) is negligible and the Hertzian relation is assumed to be valid even in the presence of adhesive forces, restricting the approach to larger grains (>mm) only. Solving Eq. (19) in Johnson et al. (1971) accordingly for the applied load $F$

$$H \xi^{3/2} = F + 3 \pi \gamma R_{\text{eff}} + \sqrt{6 \pi \gamma} R_{\text{eff}} F + (3 \pi \gamma R_{\text{eff}})^2 \quad (5)$$

yields the elastic and adhesive component of the contact force. If mutual gravity $F_{\text{grav}} = G m_1 m_2 / d^2$, where $m_1$ and $m_2$ denote the particle masses, is balanced against adhesive attraction $F_{\text{ad}} = F_{\text{grav}}$, we gain regimes of influence with respect to surface energy $\gamma$ and grain size $R$ as depicted in Fig. 1. In order to sustain a contact dominated by adhesion, $\gamma$ has to increase sufficiently with an increasing particles size since gravity becomes dominant for larger bodies. However, adhesive effects are apparent for grains up to meters in size.
\[ m_{\text{eff}} \ddot{\xi}(t) + \frac{3}{2} \frac{A H}{\sqrt{\pi}} \xi(t) + H \xi(t)^{3/2} \]
\[ - \sqrt{6\pi \gamma H R_{\text{eff}}} \xi(t)^{3/4} = 0. \]

In order of their appearance in Eq. (6), the acceleration terms signify inertia with \( m_{\text{eff}} \) denoting the reduced mass, viscous dissipation, Hertzian elasticity, and adhesion, respectively. The initial conditions are given by the impact velocity \( \xi(0) = \nu_{\text{imp}} = \ddot{\xi}(0) \) and the initial deformation \( \xi(0) = 0 \). We numerically solved Eq. (6) for various impact velocities \( \xi(0) = \nu_{\text{imp}} \) and particle sizes using material properties for ice at low temperatures: \( \rho = 10^3 \text{ kg m}^{-3} \), \( Y = 7 \times 10^9 \text{ N m}^{-2} \), \( \nu = 0.25 \), \( \gamma = 0.74 \text{ N m}^{-1} \), and \( A = 10^{-4} \) s (e.g., Chokshi et al., 1993; Brilliantov et al., 1996). \(^1\) The normal component of the deformation exhibits a damped nonlinear oscillation that crucially depends on the impact velocity as illustrated in Fig. 2.

Below a critical impact speed \( (\nu_{\text{imp}} \leq \nu_{\text{cr}}) \), a post-collisional separation is prevented and short-range forces prevail. Due to the static adhesion approach, a nonzero deformation

\[ \xi_{\text{ad}} = \left( \frac{6\pi \gamma R_{\text{eff}}}{H} \right)^{2/3} \]

remains which does not depend on the impact speed \( \nu_{\text{imp}} \). It denotes a fixed point solution of the contact dynamics and arises from an equilibrium between elastic restoring and adhesive attractive forces. \( \xi_{\text{ad}} \) ranges within \( 10^{-7} \) to \( 10^{-5} \) m for grains of mm up to tens of meters in radius. The size dependence of the dynamics will be denoted according to grain size \( R_1 = R \) and size ratio \( \mu = R_2/R_1 \).

Two possible collisional outcomes were found: restitution and aggregation. The first occurs for higher impact speeds \( (\nu_{\text{imp}} > \nu_{\text{cr}}) \), where both particles separate as indicated by \( \xi < 0 \) (gray curves in Fig. 2). The coefficient of normal restitution is...
Fig. 3. Restitution coefficient: Different impact speeds give rise to different impact scenarios: agglomeration and restitution. Both outcomes can be easily distinguished in terms of the restitution coefficient $\varepsilon_N (\nu_{imp})$ ($\varepsilon_N = 0 \Rightarrow$ agglomeration $\leftrightarrow \varepsilon_N > 0 \Rightarrow$ restitution). The adhesive viscoelastic impact model is in qualitatively good agreement with lab measurements. Impacts are overall more dissipative than in the pure viscoelastic case as indicated in Hatzes et al. (1991).

Fig. 4. Size dependence: The collision time $\tau$ increases with increasing particle sizes (upper). It is plotted for varying impact speeds $\nu_{imp}$ and different particle sizes $\mu$. A collision of cm-sized grains occurs within ms and becomes more elastic for larger grains (lower). Despite its crucial dependence on $\nu_{imp}$ (cf. Fig. 3), the restitution coefficient $\varepsilon_N$ is qualitatively insensitive to $\nu_{imp}$ with respect to its size dependence. Smaller grains stick together below a certain impact speed $\nu_{cr}$, which is smaller for larger particles (see also Fig. 5). Values applied to Higa et al. (1998) and taken as presented in Dilley and Crawford (1996, Fig. 1).

is not accounted for in the present model. The sticking forces vary widely with surface structure and impact speeds, where in this model $F_{ad}$ does not depend on either one and is at least an order of magnitude stronger. For the same reason, $\xi_{ad}$ does not depend on $\nu_{imp}$ in deviation from experimental findings. However, especially for varying surface structure and multiple collisions, $\gamma$ is widely unknown. Additionally, the dynamics with respect to $\nu_{cr}$ and its mass dependence is rather insensitive to comparative changes in $\gamma$. Previous studies (Dilley, 1993; Dilley and Crawford, 1996; Higa et al., 1998) indicated the importance of the size of colliding grains, showing more elastic contacts for larger particles than for smaller ones. The size dependence of the collision dynamics is illustrated in terms of the collision time $\tau$ (Fig. 4, upper part) and the restitution coefficient $\varepsilon_N$ (Fig. 4, lower part).

The collision time $\tau$ increases with a decreasing impact speed. It usually ranges within a few milliseconds for cm-sized

obtained as

$$\varepsilon_N = -\frac{\xi(t)}{\xi(0)},$$

where $\tau$ denotes the collision time elapsing while $\xi(t) > 0$ starting at the initial contact at $\xi(0) = 0$. The amount of dissipated energy $\Delta E_{kin} = (1 - \varepsilon_N^2)E_{kin}$ is illustrated in Fig. 3 via the restitution coefficient for varying impact speeds.

Agglomeration, i.e. $\varepsilon_N = 0$, indicates a complete dissipation of kinetic energy of the relative normal motion $\dot{d} = \dot{\xi}(\tau) = 0$, whereas restitution $\varepsilon_N > 0$ implies a post-collisional separation of both particles at $\dot{d} = -\dot{\xi}(\tau) > 0$. For a purely elastic impact ($\varepsilon_N = 1$), no kinetic energy is dissipated. Therefore, it is possible to distinguish between agglomeration and restitution in terms of the restitution coefficient $\varepsilon_N$. Within this model (cf. Fig. 3), restitutive collisions including adhesion are generally more dissipative than pure viscoelastic ones in accordance with observations by Hatzes et al. (1991).

Bridges et al. (1984) measured $\varepsilon_N$ for ice particles of 2.75 cm radius in head-on collisions at low temperatures. Assuming a pure viscoelastic contact ($\gamma = 0$) in Eq. (6) it is possible to reproduce the qualitative behavior (dotted line in Fig. 3). Numerical results may be fitted to match experimental ones (Brilliantov et al., 1996). An agglomeration of particles has actually been observed in experiments with frost covered icy grains of 2.5 cm radius (Hatzes et al., 1991; Bridges et al., 1996). Particles had different frost coatings of various thicknesses. The measured sticking velocities were around 0.03 cm s$^{-1}$ but never exceeded 0.4 cm s$^{-1}$. Results obtained in our numerical studies for the critical impact speed $\nu_{cr}$ are in good agreement with these observations. Note, that for a proper comparison of these results, $R_2 \rightarrow \infty$ has to be considered in order to account for a flat wall as one of the collision partners. Repeated collisions significantly change the surface structure by compressing the frost layer and thus strongly influence the sticking behavior (Hatzes et al., 1988), leading to an overall more elastic contact, which
grains, but abruptly reaches infinity in case of agglomeration ($v_{imp} \leq v_c$). Collisions themselves are more and more conservative for larger particles as indicated in Fig. 4. This trend toward pure elasticity also holds for contacts of different proportions ($\mu \neq 1$). Our results are in accordance with their findings as given in Higa et al. (1998) for frosted-surface ice.

Since Eq. (6) is valid for the normal component of the contact dynamics, it only partially accounts for oblique impacts. For a better understanding, future models should yield a combined description accounting for the mutual feedback of normal and tangential component. Especially the tangential feedback on the normal motion has not been considered in previous models (Brilliantov et al., 1996; Thornton, 1997). The consideration of a back-reaction of the tangential on the normal component of the collision would allow for a more elaborate study. For the time being, our model should suffice roughly to meet the major outcomes expected to occur in low-velocity contacts of ring particles.

In our numerical simulations, the collisional outcome is determined by the impact velocity on one hand and on grain sizes on the other. The mass-velocity parameter space is divided into sections of agglomeration, restitution, and fragmentation. These results are of major importance for the description of particle ensembles on a kinetic level (Spahn et al., 2004). Smaller particles (mm in size) are very likely to stick to each other or on larger ones, whereas larger particles have to have a rather small impact velocity in order to agglomerate. Nevertheless, the critical velocity for restitution to occur becomes drastically smaller for increasing masses as shown in Fig. 5. A critical velocity for fragmentation to occur is, e.g., experimentally obtained by Higa et al. (1998).

The model presented here does neither include any specific surface coatings or textures nor repeated collisions. However, it reproduces the experimentally observed collision dynamics in terms of oscillating motion and size dependence, the occurrence of sticking as well as the generally decreased restitution coefficient found by Hatzes et al. (1991). It furthermore denotes a theoretical tool to account for agglomeration and restitution. Although collisional agglomeration is possible, it is unlikely to account for reasonable growth processes. Nevertheless, it further supports the existence of regolith layers. While sticking forces come from small particles, the most mass remains within the larger ones (Tremaine, 2003).

3. Agglomerates in a tidal regime

Considering two particles of masses $m_1$ and $m_2$ in orbit about a central mass $M$, their equations of motion including pairwise interactions ($P$) read

$$m_1 \ddot{r}_1 = -\frac{GMm_1r_1}{r_1^3} + Pf[(\hat{r}_1 - \hat{r}_2)],$$
\hspace{1cm} (9)

$$m_2 \ddot{r}_2 = -\frac{GMm_2r_2}{r_2^3} + Pf[(\hat{r}_2 - \hat{r}_1)].$$
\hspace{1cm} (10)

Fig. 5. Critical velocity: $v_{cr}$ is obtained from numerical solutions of Eq. (6). The size (upper) and size ratio dependence (lower) are shown for different proportions. In general, smaller or slower grains do agglomerate while others do not stick. The gravitational escape velocity $v_{esc}$ is given as a reference in either gray lines (upper) or as a gray shaded area (lower). The influence of either adhesive processes or gravitational influence can be clearly distinguished.

Their dynamics may as well be expressed in their center of mass frame

$$\ddot{r}_c = \frac{m_1 \ddot{r}_1 + m_2 \ddot{r}_2}{m_1 + m_2}, \quad \ddot{d} = \ddot{r}_2 - \ddot{r}_1,$$
\hspace{1cm} (11)

where $\ddot{d}$ denotes the relative distance between both particles as in Eq. (2). After series expanding the central gravity around $\ddot{r}_c$ and changing to a co-moving frame, the equations of motion may be written as

$$\frac{d^2}{dt^2} \ddot{r}_c = -GM \ddot{r}_c \frac{r_c}{\ddot{r}_c},$$
\hspace{1cm} (12)

$$\ddot{d} + 2 \ddot{\Omega} \times \ddot{d} + \ddot{\Omega} \times \ddot{\Omega} \times \ddot{d} = -\Omega^2 \left( \ddot{d} - \frac{3(\ddot{d} \cdot \ddot{r}_c)\ddot{r}_c}{r_c^2} \right) + Pf[\ddot{d}].$$
\hspace{1cm} (13)
where $\Omega = \sqrt{GM/r_c^3}$ denotes the orbit frequency. Equation (13) describes the relative motion and covers all inertia forces. As shown in Section 2, the collision dynamics allows for agglomeration and restitution. Since the dynamical timescales of restituting collisions, $\tau \sim \text{ns}$ for cm-sized grains, are by orders of magnitude smaller than the average time between collisions for the saturnian ring system, particle impacts are considered as instantaneous in the following. Thus, only the collisional outcome is of importance to the dynamical evolution of a system, validating kinetic assumptions (e.g., Longaretti, 1989; Spahn et al., 2004). In case of restitution, particles are henceforth only influenced by the central body, their mutual gravity and future collisions. Agglomeration on the other hand revealed a remaining attractive bond [Eq. (3)], which has to be considered for any ongoing motion as a constant attractive particle interaction. In order to account for both collisional outcomes, particle interactions $PI(d)$ must be expressed accordingly

$$ P_{I\text{rest}}[d] = -G \frac{m_1 m_2}{d^2} e_d, \quad (14) $$

$$ P_{I\text{agg}}[d] = -\left(G \frac{m_1 m_2}{d^2} + \frac{3}{2} \pi \gamma R e_{d} d\right) e_d, \quad (15) $$

where the former refers to restitution and the latter to agglomeration. An aggregate itself moves at the center of mass velocity $\vec{v}_c$. Agglomerates exposed to a tidal shear can exist only if their bonds are strong enough. Despite a tidal disruption they might be destroyed by further perturbations such as collisions, scattering, etc. Processes considered as erosion or fragmentation together with agglomeration could ensure a steady size distribution. Instead of solving the dynamics of Eqs. (12)–(15), we would like to discuss qualitatively the stability of aggregates in a tidal environment in the following sections. To this aim, we deliberately waive the complex kinetics of the collision frequency and likelihood of an agglomerating collision, since it nevertheless remains questionable, whether aggregates based on adhesive and gravitational bonds could endure a tidal environment.

3.1. Stability of two-body agglomerates

In this section, we discuss the stability of an agglomerate held together by adhesion and mutual gravity. As in case of the Roche limit $r_{\text{Roche}}$, a critical distance $r_{\text{crit}}$ is found by analyzing particular grain configurations in orbit about a central planet. Any agglomerate, which is closer to the central body than this critical distance ($r < r_{\text{crit}}$), will inevitably be torn apart. We will hereafter refer to agglomerates that are able to resist the tidal shear as stable and those torn apart as unstable. In order to determine the functional dependence of the borderline between stability and instability, we assume two solid spherical particles of arbitrary size to have formed an agglomerate according to Section 2 and “place” it at an arbitrary orbit location. As shown in Section 2, the contact of such aggregates is static $\dot{d} = 0 = d$ and implies a constant distance $d = R_1 + R_2 - \xi_{\text{ad}} = \text{const.}$ Since $\xi_{\text{ad}}$ is of the order of $10^{-7}$–$10^{-5}$ m, it is thus negligible and Eq. (2) can be approximated by $d \approx R_1 + R_2$. The properties of a two particle agglomerate are particle radius $R = R_1$, radius ratio $\mu = R_2/R_1$, spin angular velocity $\dot{\alpha}(t)$, orientation $\alpha(t)$, and surface energy $\gamma$. According to Fig. 6 we introduce $\vec{d} = \vec{d}_e = d \cos \alpha \vec{e}_r + d \sin \alpha \vec{e}_\theta$.

For simplicity and in order to give a conservative estimate for maximal sizes of stable agglomerates, we assume $\vec{d} \perp \vec{d}$ and $\vec{d} \parallel \vec{r}$ denoting maximum tidal and centrifugal forces, where $\omega$ denotes the spin of the aggregate. Equations (13) and (15) may then be written as

$$ -\Omega^2 d(1 - 3 \cos^2 \alpha) - \frac{G(m_1 + m_2)}{d^2} \left[\dot{\alpha} + \alpha \Omega\right]d - \frac{3}{2} \pi G \mu \gamma \frac{R_{\text{eff}}}{m_{\text{eff}}} = 0. \quad (16) $$

The equilibrium distance $r_{\text{crit}}$ emanates from the orbit frequency $\Omega_{\text{crit}}^2 = GM/r_{\text{crit}}^3$ and can be obtained by solving Eq. (16). Note, $\dot{\alpha}$ denotes the spin with respect to the co-moving frame while the total spin is given by $\omega = \dot{\alpha} + \Omega$. Although $\alpha = 0$ is assumed as a snapshot, different rotation rates may be discussed. For convenience, we denote $\dot{\alpha} = k \Omega_{\text{crit}}$, where $k$ is a real number. The critical orbit distance can be written as

$$ r_{\text{crit}}^3 = \frac{GM d m_{\text{eff}}[(k + 1)^2 + 2]}{AD + SG}, \quad (17) $$

where $AD = 3/2 \pi \gamma R_{\text{eff}}$ and $SG = G m_1 m_2 d^{-2}$ refer to adhesive and gravitational particle interactions, respectively. Resolving the aggregate properties given above, Eq. (17) becomes

$$ r_{\text{crit}}^3 = \frac{24G M d R^3 [1 + \mu^2] [(1 + 1)^2 + 2] (1 + \mu^3) [27 \gamma + 32 \pi G \mu^2 R^3]}{(1 + \mu^3) [27 \gamma + 32 \pi G \mu^2 R^3]}, \quad (18) $$

which further reduces in case of equally sized grains ($\mu = 1$) to

$$ r_{\text{crit}}^3 = \frac{48 G M d R^3 [2 + (k + 1)^2]}{27 \gamma + 16 \pi G R^3}. \quad (19) $$

The critical distance $r_{\text{crit}}$ may as well be obtained as an effective value according to

$$ \frac{1}{(r_{\text{crit}})^3} = \frac{1}{(r_{\text{crit}}^3)^{\text{SG}} + (r_{\text{crit}}^3)^{\text{AD}}}. \quad (20) $$
where

\[
(r_{\text{crit}}^{SG})^3 = \frac{3M[(k + 1)^2 + 2]}{4\pi \rho} (1 + \mu)^3, \tag{21}
\]

\[
(r_{\text{crit}}^{AD})^3 = \frac{8GM\rho[(k + 1)^2 + 2]}{9\gamma} \mu^2 (1 + \mu)^2 \bigg/ (1 + \mu)^3 \quad R^3. \tag{22}
\]

Any additional attractive force can thus easily be implemented. Equation (21) denotes the equilibrium distance, if mutual gravity is considered alone. It only depends on the agglomerate proportions but does otherwise not imply any critical agglomerate sizes. It is important to mention, that the inclusion of adhesion allows for a size dependent critical distance to the central planet expressed by Eq. (22). The critical distance as well as the adhesive bond does not depend on the mechanical properties of the grains. Fig. 7 illustrates \( r_{\text{crit}} \) in case of two radially-aligned and synchronously rotating particles in contact for the saturnian system and material parameters as applied in Section 2.

The critical range \( r_{\text{crit}} \) is highly sensitive to agglomerate sizes and clearly arises as an effective value of a pure adhesion and gravitational particle interaction [Eq. (20)]. The asymptotic gravity limit coincides with the outer edge of the B-ring for particles of tens of meters in size and \( \mu = 1 \). It denotes the asymptotic value for larger bodies \((R \to \infty)\) that are liable to gravity. Adhesion dominates in case of smaller grains \((R < 10 \text{ m})\) where \( r_{\text{crit}} \approx r_{\text{AD}} \). As soon as smaller particles stick on larger ones \((\mu \ll 1 \text{ or } \mu \gg 1)\), agglomerates of such kind would be stable even as close to Saturn as the D-ring. In the lower part of Fig. 7, the obvious symmetry with respect to \( \mu \) is clearly visible. Aggregates become unstable for \( 0.1 \leq \mu < 10 \). The remaining parameter space refers to stable radially-aligned agglomerates.

In case of nonsynchronously spinning aggregates \((k \neq 0)\) additional centrifugal forces tear at an adhesive bond. Fig. 8 shows the influence of different spin configurations on \( r_{\text{crit}} \).

Nonspinning aggregates \((\omega = 0, k = -1)\) appear to be the most stable ones. Any rotation shifts \( r_{\text{crit}} \) farther outward. Only smaller agglomerates can sustain themselves at high rotation rates. Rotation rates of ring particles have been investigated in previous studies [Salo, 1987 (nongravitating particles); Salo, 1995 (gravitating particles of identical size); Ohtsuki, 2005 (gravitating particles with size distribution)] and showed a decreasing spin rate with increasing particle sizes. Spin rates of moonlets embedded in planetary rings were discussed in [e.g., Morishima and Salo, 2004; Ohtsuki, 2004]. It has further been shown, that not only the average rotation rate but also the spin dispersion is of major importance to the dynamics within ring systems. But implementing a spin dispersion \( \sim 1/(R(1 + \mu)) \) (Salo, 1987) has no significant influence on \( r_{\text{crit}} \). Changes in \( R, \mu, \text{ or } \omega \) remain within 0.05 \( R_S \).

Weidenschilling et al. (1984, pp. 376–377) gave expressions for the distance where the net attraction between two spherically, radially-aligned particles in contact is zero as following: equally sized bodies in synchronous rotation [their Eq. (2)] and not rotating [Eq. (3)]; small particle resting on a larger one in synchronous rotation [Eq. (4)] and not rotating [Eq. (5)]. Applying these parameters to Eq. (17), we find a good agreement with our results as shown in Fig. 9.

Each thick horizontal line corresponds to one particular set of parameters, which is reproduced as an asymptotic limit in case of large \( R \). Longaretti (1989) deduced a critical density in case of gravitational attraction only and that aggregates with \( \mu \neq 1 \) effectively attract each other, noting an indirect dependence on the radial distance of these findings.

Agglomerates are most fragile if they consist of equal sized particles, since first of all small particles stick more easily on larger ones (indicated by a comparably high critical velocity, see also Fig. 5) and in addition, the stability zone for \( \mu \neq 1 \) (see Fig. 7) is definitely broader. Furthermore, the center of mass of each constituent for \( \mu \approx 1 \) is located outside of the gravitational range of the other particle, whereas a small particle resting on a larger one is itself well inside the Hill-sphere of the latter (Ohtsuki, 1993). Therefore, the agglomeration of smaller par-
ticles (μ ≠ 1) appears always possible, further promoting the existence of regolith layers. Their thickness will strongly depend on the size of the parent body and thus indirectly on the radial distance from the central planet. Regolith layers strongly determine the elasticity of a future impact. Differences in the elasticity of the average collision with respect to the orbit location are thus to be expected.

Nevertheless, stability seems to be highly pronounced compared to observations implying agglomerates of tens of meters in size. Hatzes et al. (1991) obtained maximum sizes for N-body agglomerates in a tidal regime of about R = 6 m for 10 cm and R = 70 m for 1 meter-sized spherical particles. But since no other perturbing or erosive processes, such as, e.g., further collisions, have been considered, results presented here denote the maximum particle sizes for a given location. Including all these processes, grain sizes are expected to be much smaller.

3.2. Collisional stability estimate of two-body agglomerates

As for the stability discussion in Section 3.1 we give a rough estimate for the effects of including erosive processes. To this aim, we check the bond strength (adhesive and gravitational) against the impact energy of yet another body. A sufficient break-up energy $E_{\text{break}} = Q M_{\text{coll}}$ is required to ensure the destruction of the agglomerate, where $Q$ denotes the critical specific energy per unit mass (Durda et al., 1998), where $M_{\text{coll}} = 4\pi \rho R^3_{\text{coll}}/3$ denotes the mass of the impacting body with radius $R_{\text{coll}}$. This critical specific energy accounts for the internal strength (elastic energy of deformation, number and strength of internal bonds between constituents, etc.) and the gravitational binding energy. The impact energy must exceed this internal energy to destroy the target. Agglomerates discussed in this paper are built in a rather simple way, because they consist of a single bond, which has a bond energy of $E_{\text{break}} = E_{\text{bond}} = E_{\text{ad}} + E_{\text{grav}}$. The adhesive energy is determined by

$$E_{\text{ad}} = \int_0^{\xi_{\text{ad}}} F_{\text{ad}}(\xi) \, d\xi = \frac{36}{7} (6\pi^2)^{1/3} \left[ \frac{\gamma^5 R^4_{\text{eff}}}{E^2_{\text{eff}}} \right]^{1/3},$$

where $\xi_{\text{ad}}$ was given in Eq. (7) and the gravitational potential reads $E_{\text{grav}} = G m_1 m_2 d^{-1}$. Although impact velocities are as high as the escape velocities of the largest ring boulders (Ohtsuki, 1993), we assume $v_{\text{coll}} = 3d \Omega/2$, for simplicity. In doing so, we indicate the stability changes since higher velocity impacts will result in even more destructive encounters. As in Section 3.1, $r_{\text{crit}}$ emanates from $\Omega_{\text{crit}}$ and reads

$$r_{\text{crit}}^3 = \frac{9 G M_S}{8} \frac{m_{\text{eff}} M_{\text{coll}}}{(E_{\text{ad}} + E_{\text{grav}})(m_{\text{eff}} + M_{\text{coll}})}.$$
In Fig. 10 a limiting distance for agglomerates of equally sized constituents is given for different sizes of an impacting third body.

The stability region is by far not as pronounced as in Section 3.1. For the inner rings the maximum sizes are within the range of decimeters. Smaller agglomerates are not as easily broken as larger ones, since adhesive bonds are stronger in this case (cf. previous sections). The tendency, that smaller particles reside closer to the planet than larger ones is retained. The main influence on grain sizes appears to arise from their mutual collisions and not from the tidal shear, as also Longaretti (1989) noted. Our assumptions underestimate the impact energy, thereby leading to larger agglomerates than to be expected but providing a rough upper limit. Anyhow, all our calculations in Sections 3.1 and 3.2 should be taken as estimates serving as a guideline for further, more detailed studies.

4. Conclusions

Presenting an adhesive viscoelastic collision model, we are able to account for agglomeration and restitution of solid grains ranging in size from millimeters up to tens of meters. Basic features and the qualitative dynamics of icy grains at low temperatures are reproduced. We obtain a size dependent critical velocity for restitution to occur that is about a few cm/s for cm-sized grains. Based on the outcome of a binary collision only, $v_{cr}$ denotes a crucial input parameter for further kinetic studies of the size and velocity distribution. A solution accounting for agglomeration, restitution, and fragmentation is to be expected within the estimates for agglomerate sizes presented here. Results are applicable to rubble piles, dynamical ephemeral bodies, and grains of low internal strength, but the effective strength of $N$-body agglomerates based on adhesive bonds has yet to be studied.

Regolith layers may be very common since collisional sticking and stability are enhanced for $\mu \ll 1$, $\mu \gg 1$. Thus, these can as easily be established as destroyed. Shattering impacts may deliver the regolith dust temporarily into orbit, which eventually will be reaccumulated. This could provide the base to account for a vivid size distribution dynamics within the F-ring. The combination of agglomerating and fragmenting dynamics based on collision dynamics implies that small particles can denote the essence of fast dynamical processes. A transient dynamics could be the result without leading to an effective growth of new agglomerates. This could be detected in the form of brightness fluctuations or general patterns of the ring itself, thereby indicating the number of moons, and would endorse observations made and models proposed (e.g., Poulet et al., 2000a, 2000b; Barbara and Esposito, 2002).

Many authors concerned about the grain sizes in planetary rings have considered only gravity as a binding force. But these critical limits as the Roche criterion could merely determine orbital distances and thus denote only a rough indication. No dependence of the sizes in terms of the distance of the agglomerate’s orbit could be derived in this way. Only the combined consideration of adhesion (or any internal strength) and mutual gravitation allows for a critical radial distance to the central body implicating a grain size dependence as given in Eqs. (17)– (19). We found a general trend for agglomerates to be smaller toward the planet and becoming larger at further distant orbit locations as observed by the Cassini mission (VIMS-experiment). The given examples restrict agglomerate sizes to decimeters for the inner ring system and thus coincides with observations done by French and Nicholson (2000), where sizes range between a few centimeters and a few meters.

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References


