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Moonlets wandering on a leash-ring

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ABSTRACT
Since the *Voyager* flybys, embedded moonlets have been proposed to explain some of the surprising structures observed in Saturn's narrow F ring. Experiments conducted with the *Cassini* spacecraft support this suggestion. Images of the F ring show bright compact spots, and seven occultations of stars by the F ring, monitored by ultraviolet and infrared experiments, revealed nine events of high optical depth. These results point to a large number of such objects, but it is not clear whether they are solid moonlets or rather loose particle aggregates. Subsequent images suggested an irregular motion of these objects so that a determination of their orbits consistent with the F ring failed. Some of these features seem to cross the whole ring. Here we show that these observations are explained by chaos in the F ring driven mainly by the 'shepherd' moons Prometheus and Pandora. It is characterized by a rather short Lyapunov time of about a few hundred orbital periods. Despite this chaotic diffusion, more than 93 per cent of the F-ring bodies remain confined within the F ring because of the shepherding, but also because of a weak radial mobility contrasted by an effective longitudinal diffusion. This chaotic stirring of all bodies involved prevents the formation of 'propellers' typical of moonlets, but their frequent ring crossings explain the multiple radial 'streaks' seen in the F ring. The related 'thermal' motion causes more frequent collisions between all bodies which steadily replenish F-ring dust and allow for ongoing fragmentation and re-accretion processes (ring recycling).

Key words: methods: numerical – celestial mechanics – planets: rings – planets and satellites: general.

1 INTRODUCTION

The complex structure of Saturn's F ring, first revealed by *Voyager* images, has often been attributed to the combined gravitational action of the 'shepherd' moons, Prometheus and Pandora, and numerous hypothetical moonlets (Showalter & Burns 1982; Lissauer & Peale 1986; Spahn & Wiebicke 1989; Kolvoord, Burns & Showalter 1990; Murray, Gordon & Giuliatti 1997). Such small moons (kilometre-sized) escaped detection by the *Voyager* cameras, but in 1995, during the crossing of the Earth through Saturn's ring plane (RPC95), observations with the *Hubble Space Telescope* (*HST*) and the European Southern Observatory (ESO) revealed such objects orbiting close to or within the F ring (Bosh & Rivkin 1996; Nicholson et al. 1996; Poulet et al. 2000) Moreover, in 2004, while approaching Saturn, the cameras of the *Cassini* spacecraft caught a small object, 4–5 km in size (provisionally named S/2004 S3), near the outer edge of the F ring. Subsequent images only about 5 h later showed it orbiting interior to the F ring – the object must have crossed the ring. The *Cassini* imaging team has found even more such objects (Fig. 1) which all were difficult to track (Porco et al. 2005a; Murray et al. 2005; Porco et al. 2005b).

This complexity of motion frustrated the derivation of orbital elements compatible with the F ring (Murray et al. 2005). Furthermore, starlight occultations by the F ring observed with the Ultra-Violet Imaging Spectrometer (UVIS) and the Visible and Infrared Mapping Spectrometer (VIMS) on *Cassini* provided hints of a large number of moonlets or agglomerates (10^5 bodies; optical depth 10^{-3}) larger than 600 m in size (Meinke, Esposito & Colwell 2006; Esposito, private commnication). These findings raise questions. Why do such moonlets not create 'propellers' or gaps in the F ring as observed in the A ring (Tiscareno et al. 2006)? What forces these objects to cross 'suddenly' the main strands of the F ring, and do they 'cut' structures in the F-ring strands during these crossings? In this Letter we address these questions by analysing the dynamics of bodies in the F ring.

2 CONFINED CHAOTIC MOTION

An unexpected observation served as an initiation of our model. During RPC95, *HST* discovered that Prometheus had a lag of about 20° behind relative to its predicted longitude (Bosh & Rivkin 1996; Nicholson et al. 1996) while Pandora was found to travel ahead of its expected position (McGhee et al. 2001; French et al. 2003). Both longitude discrepancies are comparable in magnitude and have opposite signs (French et al. 2003), suggesting a complex coupled

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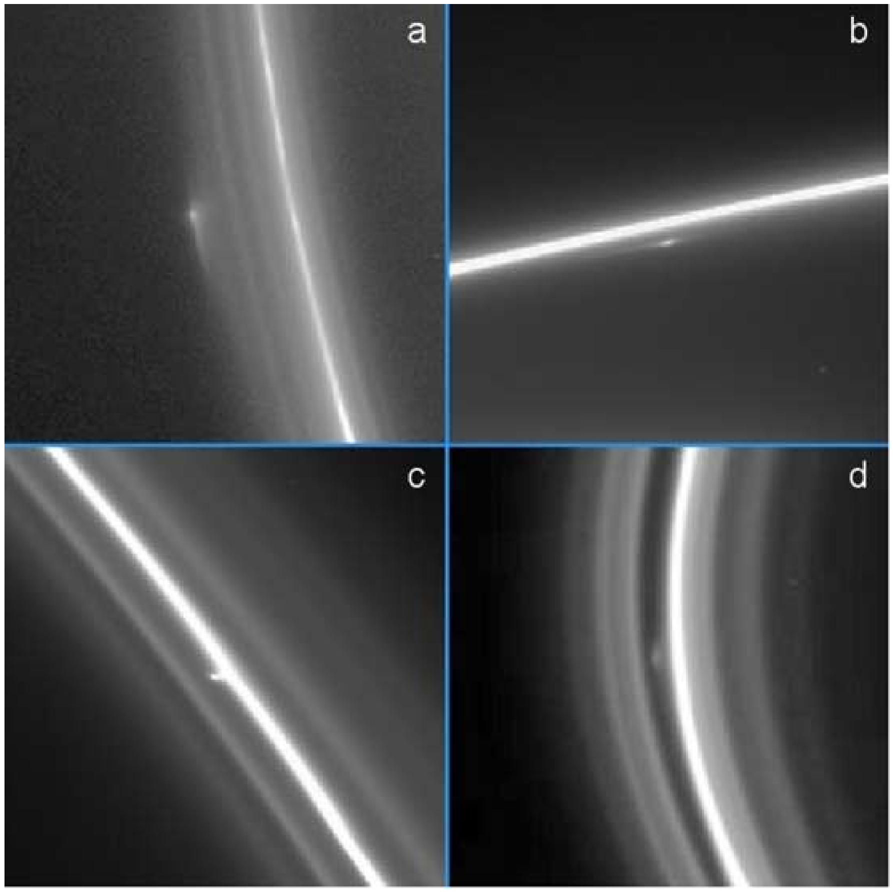

Figure 1. *Cassini* images of Saturn's F ring: a set of four images of Saturn's F ring taken by the *Cassini* Imaging Science Subsystem. (a) Image taken on 2005 June 21 showing an object in the outer ringlets of the F ring. (b) Image taken on 2005 June 29 showing a bright feature within the inner ringlets of the F ring. (c) Image taken on 2005 August 2 showing a feature that may be S/2004 S3. (d) Image taken on 2005 April 13 showing an object that is near the main strand of the F ring. (Courtesy NASA/JPL-Caltech/ESA)

motion of the two 'shepherd' moons. In conclusion, chaotic dynamics of both satellites have been found (Dones et al. 2001; Goldreich & Rappaport 2003a,b) to unravel the mystery. The chaotic motion occurs already if the two 'shepherds' are considered alone (Goldreich & Rappaport 2003b), and advanced studies (Cooper & Murray 2004; Renner & Sicardy 2005), including the perturbations of other Kronian satellites, have confirmed this result.

Is such a chaotic motion generally characteristic of all bodies – ring particles and moonlets – revolving between both chaotic 'shepherds' Prometheus and Pandora? In order to find an answer we have performed test-particle experiments taking into account the gravitational influence of both 'shepherds' and Saturn, including the harmonics J_2, J_4 and J_6 due to its oblateness. Thousands of test-particles, homogeneously distributed and having initial conditions compatible with the orbital elements of the F ring, have been integrated to follow their orbital motion. We have performed numerical integrations via the Bulirsh–Stoer method using the package MERCURY (Chambers 1999). The moonlet has a diameter of 5 km and a density of $0.65\,\mathrm{g\,cm^{-3}}$. The initial conditions of the satellites are derived from the *Voyager* data (Evans 2001) and the moonlet has initial conditions similar to the geometric orbital elements (Borderies-Rappaport & Longaretti 1994; Renner & Sicardy 2006) of the F ring. We have considered a sample of 1080 moonlets distributed at every single degree in mean anomaly and at three different values of semi-major axis: the nominal value of the semi-major axis of the F ring (a_{ring}), and $\pm$ 50 km from this value.

The simulation covers the period from *Voyager*'s passage in 1981 until the arrival of the *Cassini* spacecraft at Saturn in 2004. It should be noted that our test-particle simulations are valid for all bodies driven by gravitational forces – grains ranging in size from millimetres up to the moonlets of interest.

A chaotic behaviour has been found for all particles. A typical erratic evolution of the semi-major axis of a particle is presented in Fig. 2. In order to quantify the character of the particle motion we have calculated the largest Lyapunov exponent (Benettin, Galgani & Giorgilli 1980), γ, where chaos is implied by $\gamma > 0$. In order to calculate the value of γ, a subset of 324 test-particles (distributed homogeneously every 10° in true anomaly, *f*, and every 50 km in semi-major axis, *a*, around the centre of the F ring $\pm$200 km) has been numerically integrated for 10^6 orbital periods. All particle trajectories have been found to be chaotic with an e-folding time for divergence of nearby orbits (Lyapunov time) $t_{\mathrm{L}} = \gamma^{-1} \approx 100T$ (*T* is the orbital period).

All trajectories show 'sudden jumps' of 10–100 km in the moonlet's semi-major axis in only a fraction of an orbital period (Fig. 2, right-hand panel). This is exactly what has been observed with the *Cassini* cameras in the case of the object S/2004 S3. In order to identify the properties of those 'jumps', a histogram of the largest semi-major axis changes is shown in Fig. 3. A mean maximum distance of $\approx$100 km can be suggested for the ensemble, whereas about 50 per cent of the particles experience semi-major axis excursions smaller than 50 km over the whole integration period. A minority

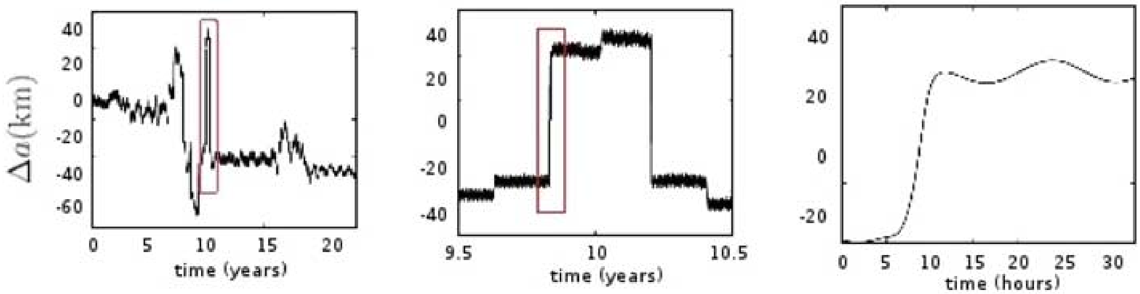

Figure 2. Example of the typical temporal evolution for the semi-major axis of a moonlet. The system was numerically integrated from 1981 to 2004. Left-hand panel: the whole length of the integration, with the time-scale in years. The evolution is chaotic as indicated by the Lyapunov characteristic exponent. Centre panel: a zoom of the left-hand panel. This part corresponds to the largest variation in semi-major axis during the whole integration. Right-hand panel: a zoom of the centre panel, with the time-scale in hours. The semi-major axis of the moonlet increases by more than 50 km in less than 5 h, about one-fourth of its orbital period.

of the particles may temporarily leave the ring, but the majority of them remain in the F-ring area. Only a few particles (6.7 per cent) reach variations in semi-major axis of several hundred kilometres, escaping the region between Prometheus and Pandora. Dissipative collisions and the self-gravity of the ring may reduce the excursions of the moonlet's orbital radius further.

All simulations have shown that ring particles as well as moonlets are relatively mobile in diffusing throughout the F ring. This mobility, however, is quite different in different spatial directions because the Lyapunov characteristic exponent (LCE) is composed of contributions from all components of the phase space (Benettin et al. 1980). In order to identify which component is responsible for the major contribution of the LCE, we have computed the radial and azimuthal contributions separately. A rather small radial component of the LCE (Lyapunov time $\gamma^{-1} \approx 10^6 T$) is contrasted by a large azimuthal component of the LCE with a Lyapunov time of the order of about $\gamma^{-1} \approx 100\,T$, suggesting a rather large azimuthal chaotic mobility. On the other hand, all F-ring bodies stay radially almost where they are – apart from the 'sudden' radial 'jumps' (see Fig. 2, right-hand panel). Thus the 'shepherding' by Pandora and Prometheus and the relatively small radial diffusion ensure an effective confinment of all ring particles in the F-ring region.

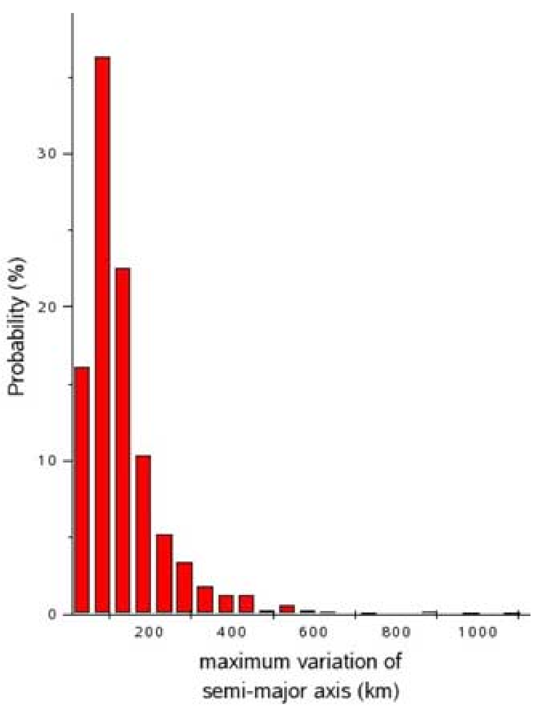

Figure 3. Histogram of the largest change in semi-major axis. The distribution of the maximal change in a is presented for the complete ensemble of test moonlets. More than 80 per cent of the moonlets show variations smaller than 200 km.

3 FEATURES IN THE RING

This chaotic mobility of all F-ring bodies, however, will be decisive for the complex structures observed there. First, the related particle diffusion and the 'sudden jumps' of the moonlets effectively prevent the creation of the characteristic 'propeller' structures (Spahn & Sremcevic 2000; Seiß et al. 2005). The generation of the latter requires two conditions: a relatively stable, almost circular orbit of the moonlet and a relatively moderate spatial diffusivity $D\,t_D \approx \langle \Delta \boldsymbol{r}^2 \rangle < D^2$ of the ring material. D is the moonlet diameter (or Hill scale $h \sim D$) and $L = \sqrt{\langle \Delta \boldsymbol{r}^2 \rangle}$ denotes the mean diffusion distance that ring particles migrate in the time t_D. The propellers need a few orbital periods $t_D \approx (2 \ldots 3)T$ to be formed by the moonlet (Spahn & Sremcevic 2000; Seiß et al. 2005), but from our simulations a velocity dispersion of the order of metres per second has been obtained, leading to $L > D$. Thus the 'propellers' are blurred out right from the beginning because the embedded moonlets are too small and the particle ensemble is too 'hot'.

On the other hand, the rather frequent radial crossings of the F-ring strands by the moonlets cause radial 'streaks' which have been observed in a few narrow ringlets (Bosh & Rivkin 1996; Nicholson et al. 1996).

We have performed numerical simulations in order to verify such features. Since we are interested in the effect produced in a ring of particles by a moonlet that crosses the ring, an analysis of this perturbation can be carried out in the neighbourhood of the moonlet. 45 000 ring particles have been randomly distributed in a 'box', radially limited in the range $a_{\rm ring} \pm 100$ km, and azimuthally limited in an angle of 6°. Simple boundary conditions have been used to return any particles that leave the box in the azimuthal direction, since only a small portion of the orbit has been integrated. When the distance between the particle and the moonlet is about the radius of the satellite, the particle is removed and a new particle starts at the edge of the box.

Fig. 4 shows a snapshot of a simulation of a moonlet crossing of a band composed of test particles. The process takes a half-orbital period $T/2$ and produces a radial 'streak' containing particles dragged out of the particle annulus. Indeed, the frequent occurrence of such

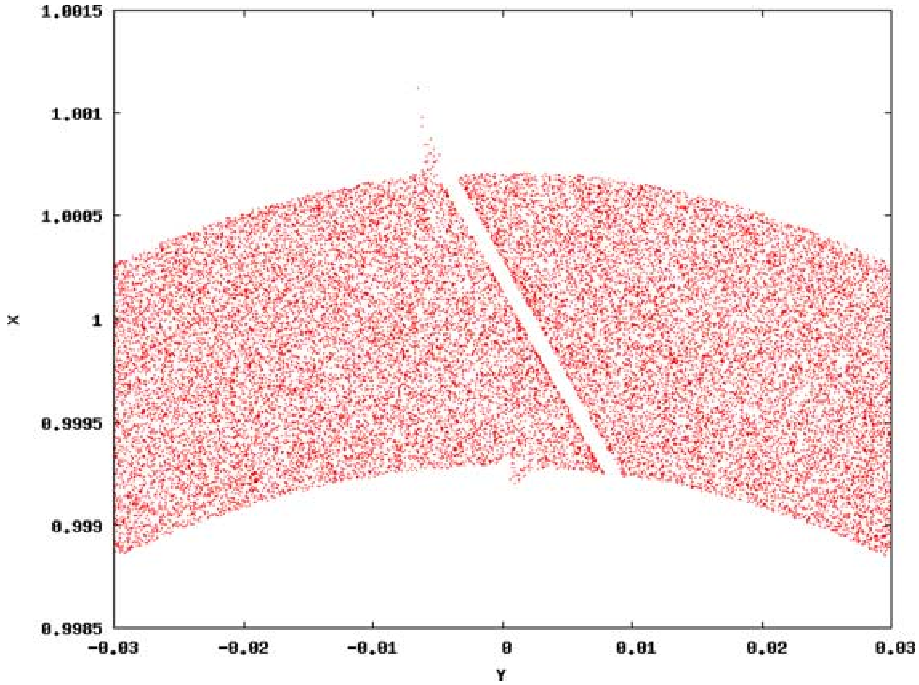

Figure 4. Snapshot of a density 'streak' caused by the short-term radial crossing of a moonlet over a narrow particle ring. The moonlet takes about $0.25T$ to cross the ring. The gap starts to open about $0.5T$ after the moonlet crosses the ring. The gap takes about $0.3T$ to be completely open and another $0.3T$ to be completely closed.

features in the F ring points to a rather large number of such moonlets or 'rubble piles' as concluded from the UVIS occultations (Meinke et al. 2006).

4 FINAL COMMENTS

We have shown that moonlets (and ring particles) 'diffuse' chaotically across the F ring. Sometimes 'sudden jumps' of F-ring moonlets cut radial 'streaks' in the bright F-ring strands. 'Propellers' may not form because of too large a mobility of the ring material compared with the size of the moonlet. In rare cases particles and moonlets may leave the F ring owing to their chaotic random walk. The short-term lively appearance of any structures – 'kinks, braids and clumps' – concluded from the *Voyager* and *Cassini* images is also explained by the mobility of the F-ring moonlets or particle aggregates. Collisions between them or with larger ring grains in the main F-ring strands are supported by the chaotic diffusion, which in turn is decisive for the supply of dust in the F ring. Thus one consequence of the chaotic dynamics, demonstrated in this work, is an ongoing fragmentation and re-accretion of the putative moonlets or agglomerates, which in this way replenish and re-cycle the F-ring material (Esposito et al. 2005), giving the ring its lively dynamic appearance.

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