Universität Potsdam
Celestial Mechanics
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1. Exercise (12.11.19)

Problem 1.1

Which of the two bodies, Earth and Sun, attracts the Moon stronger? The ratio of the masses is 1 : 330000, the ratio of the distances is 1 : 400.

Problem 1.2

Two reference systems S and S' are identical at $t = 0$. Then, they still share the same point of origin but S' rotates with a constant angular velocity $\vec{\omega}$ with respect to S. Derive the following relation:

$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}}_{\text{seen from } S} = \underbrace{\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)'}_{\text{seen from } S'} + \vec{\omega}\times \tag{1}$$

Hints:

1. A position vector $\vec{r}$ can be expressed by $\vec{r} = \sum_i r_i \hat{e}_i = \sum_i r'_i \hat{e}'_i$ in S and S', respectively.
2. The velocity as seen from S is $\dot{\vec{r}} = \sum_i \dot{r}_i \hat{e}_i = \sum_i \left(\dot{r}'_i \hat{e}'_i + r'_i \dot{\hat{e}}'_i\right)$, whereas it is $\dot{\vec{r}}' = \sum_i \dot{r}'_i \hat{e}'_i$ as seen from S'.
3. To determine the change of the unit vectors $\hat{e}'_i$ as seen from S, consider an arbitrary vector $\vec{a}'$ (i.e. $\hat{e}'_i$) that is fixed in S' and calculate its change in the time interval $[t, t + \mathrm{d}t]$ as seen from S.

Problem 1.3

Consider the vector space of (real) polynomial functions with $\{1, x, x^2, x^3, \ldots\}$ as a basis and $\langle f(x), g(x)\rangle = \int_{-1}^{1} f(x)g(x)\mathrm{d}x$ as scalar product (*optional:* remember the properties of a scalar product and show that $\langle\cdot,\cdot\rangle$ fulfils them).
Find the first three vectors of an orthogonal basis $P_n(x)$, such that $\langle P_n, P_m\rangle = 0$ for all $n, m \in \{0, 1, 2\}$ with $n \neq m$ and $P_n(1) = 1$. What is the name of these polynomials and what method did you use?
Show that $(1 - 2xz + z^2)^{-1/2} \approx \sum_{n=0}^{3} P_n(x) z^n$.

Problem 1.4

Show that $\vec{\nabla} \cdot \vec{r}/r^3 = 0$ for $r > 0$. Find a way to calculate $\int_V \vec{\nabla}\cdot\frac{\vec{r}}{r^3}\,\mathrm{d}V$ for a volume V that includes the origin.