Universität Potsdam
Celestial Mechanics
Daniel Schirdewahn

Institut für Physik & Astronomie
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dschirde@uni-potsdam.de

0. Exercise (21.10.19)

Problem 0.1

The Cartesian coordinates of the plane x, y are given by the (polar) elliptic coordinates u, v as

$$\begin{pmatrix} x \\ y \end{pmatrix} = c \begin{pmatrix} \cosh u \cos v \\ \sinh u \sin v \end{pmatrix} \quad , \tag{1}$$

where $u \in [0, \infty)$, $v \in [0, \pi)$ and c is a constant.

(a) Find the unit vectors $\vec{e}_u$ and $\vec{e}_v$ and show that the are orthogonal.

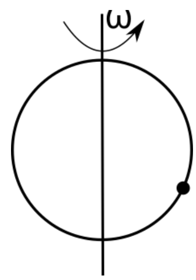
(b) Show, that the coordinate lines of u and v are ellipses and hyperbolas. What is the meaning of c?

Problem 0.2

Let x_i be Cartesian coordinates and $x_i = x_i(q_\nu)$, with curvilinear coordinates q_ν and $i, \nu = 1, 2, 3$.
Show that $\nabla = \frac{\vec{e}_\nu}{g_\nu} \frac{\partial}{\partial q_\nu}$.
Find $\nabla \cdot \vec{A}$ in spherical coordinates, where $\vec{A}$ is a vector field.
Remember: The differential operator 'nabla' ∇ can be expressed as $\nabla = (\partial/\partial_x, \partial/\partial_y, \partial/\partial_z)^T$ and the scale factor is $g_\nu = \left\| \frac{\partial \vec{r}}{\partial q_\nu} \right\|$

Problem 0.3

Imagine a metal ring with a small body sliding frictionless along the wire. The ring is rotating (along an axis lying within the rings plane and through its center of gravity) with a constant angular velocity ω. Gravity shall be acting downwards along the rotation axis.
What forces are acting on the body? Find the equations of motion for the body's position $\vec{r}$. Describe the motion of the body.



Additional 'brain twisters': At which positions on the ring could the body be at rest? What would happen, if the body's initial conditions lie close to that '*fixed points*' (small velocity / small displacement)?