Spahn *et al.* reply: In their Comment on paper [1], Pöschel and Schwager [2] criticize the explicit formulation of the dependency of the coefficient of restitution on the relative impact velocity $\epsilon(v_{\rm imp})$. We have used a power law fit-function $\epsilon \propto (v_{\rm imp} + v_*)^\beta$ which is for $\beta = -\frac{1}{4}$ (all velocity units are in cm s^{-1}) in fairly good agreement with the numerical solution of the equation of the collisional dynamics (Eq. (18) in [3]) as well as with results of laboratory measurements with icy spheres [4].

After our manuscript was in press, Schwager and Pöschel [5] solved this dynamical equation analytically resulting in

$$\epsilon(v_{\rm imp}) = 1 - \gamma_1 v_{\rm imp}^{1/5} + \gamma_2 v_{\rm imp}^{2/5} + \sum_{i=3}^{\infty} (-1)^i \gamma_i v_{\rm imp}^{i/5}, \qquad (1)$$

which is certainly a commendable improvement of the knowledge about contact dynamics. The coefficients γ_i in (1) are themselves given in the form of a series [5], and it takes considerable effort to calculate them analytically. However, in [5], only γ_1 and γ_2 are given explicitly. Any incomplete series, hence, has a limited range of validity. Considering only the first three terms, as in Fig. 1 of the Comment, leads to a quadratic expression in $v^{1/5}$ which obviously fails for large $v_{\rm imp}$. However, we agree with the statement in the Comment that our fit deviates from the exact solution of the equation of collisional dynamics for $v_{\rm imp} \to 0$.

Therefore, we discuss the consequences of $\epsilon(v_{\rm imp})$ as given by (1) to our results: The basic relations for the instability of Eqs. (4) and (6) in [1] remain unchanged. The critical wave number k_c—given by Eq. (5) in [1]—can be written more generally:

$$k_c = \left\{ \frac{-\gamma_0}{\kappa_0(\frac{\partial P}{\partial \rho}|_0)} \left[\frac{1}{\rho_0} \frac{\partial P}{\partial T}\bigg|_0 + \frac{\partial P}{\partial \rho}\bigg|_0 \frac{\partial \ln e}{\partial T}\bigg|_0 \right] \right\}^{1/2}, \qquad (2)$$

[with $e = 1 - \epsilon(T)^2$] showing qualitatively the same tendencies for any relation of $\epsilon(v_{\rm imp})$. For $T_0 \to 0$, k_c gets zero in any case, corresponding to vanishing clusters. This fact does not depend on the collision law.

As mentioned in the Comment, our fit for $\epsilon(v_{\rm imp})$ and the analytical solution (1) agree well for larger impact velocity. Therefore, the numerical results in [1] concerning the driven case remain unaffected.

We admit that (1) is a solution (provided all γ_i are known) for the contact dynamics of ideal nonadhesive spheres and, hence, should be applied when considering granular gases at low granular temperature.

When dealing with "realistic" materials, as, e.g., found in planetary rings, however, another problem arises. In this context, Chokshi *et al.* [6] have theoretically modeled the microphysics of the coagulation and collision processes and found that spherical grains stick completely provided the impact velocity drops below a certain critical value $v_{\rm cr} \propto R^{-5/6}$ (R—radius of the particles). For instance, centimeter-sized grains (silicate) stick at $v_{\rm imp} < 0.01$. For icy particles, which had been of interest in [1], these critical values are considerably larger. In this range, the collision dynamics must be extended by an attractive force; i.e., neither our fit nor (1) describes physical reality correctly. Those effects will be the subject of ongoing work.

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Received 12 September 1997 [S0031-9007(98)06487-4]
PACS numbers: 83.50.–v, 05.20.Dd, 47.55.Kf, 96.35.Cp

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