Author Correction: Bayesian deep learning for error estimation in the analysis of anomalous diffusion

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Correct references are:

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