University of Potsdam Institute of Physics and Astronomy
Lecture Stochastic Processes (SS 2018)
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Problem Set 7

(discussion on June 21st)

Drift and subdiffusion to an absorbing boundary (continued)

At time $t_0 = 0$ a density $Q_\alpha(x, t_0) = \delta(x - x_0)$ is created at a position x_0. We consider a subdiffusive random process in the presence of an absorbing boundary at the origin, with a bias $v_\alpha < 0$ and evolving according to the fractional diffusion equation

$$\partial_t Q_\alpha = {}_0D_1^{1-\alpha} \left(-v_\alpha \partial_x Q_\alpha + K_\alpha \partial_x^2 Q_\alpha \right). \tag{1}$$

We can interprete the subdiffusive process with density Q_α as a subordination of the normal drift-diffusion process with absorbing boundary and density Q_1.

$$Q_\alpha(x, u) = \int_0^\infty Q_1(x, s) \mathcal{E}_\alpha(s, u) ds \tag{2}$$

How are the mean $\langle x \rangle_{Q_\alpha}$, the survival probability $\mathcal{S}_\alpha$ and the first passage time density p_α related to their counterparts of the non-subordinated, normal drift-diffusion process (in Laplace space and in the time domain)? Calculate or recall $\langle x \rangle_{Q_1}$, $\mathcal{S}_1$ and p_1 and plot the subordinated quantities in double logarithmic scales, e.g. for $\alpha = 1/2$.